

AERO

MATHEMATICS CURRICULUM FRAMEWORK K-8 STANDARDS AND PERFORMANCE INDICATORS



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|--------------------------|------------------------|-------------------------------|---------------------------------|--|---|--|
| | | | | | Content Standards | |
| | | | | | Numbers and Operations | |
| | | | | | Patterns, Functions, and Algebra | |
| | | | | | Spatial Relationships , Geometry, and Logic | |
| | | | | | Measurement | |
| | | | | | Data Analysis | |
| Process Standards | PROBLEM SOLVING | MATHEMATICAL REASONING | MATHEMATICAL CONNECTIONS | MATHEMATICAL COMMUNICATION AND REPRESENTATION | The mathematical processes provide the framework for teaching, learning, and assessing in mathematics at all grade levels. Instructional programs should be built around these processes. | |
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AERO MATHEMATICS CURRICULUM FRAMEWORK K-8 STANDARDS AND PERFORMANCE INDICATORS

Teaching Mathematics for the 21st Century

We need citizens who can problem solve and think critically to compete in an ever-changing technological and global society. We must produce students who are capable of becoming life-long learners and successful citizens in a global market place. Therefore, students must develop a deep understanding of mathematical concepts and possess a strong foundation of number sense in order to become proficient in mathematics.

Every teacher of mathematics has an individual goal to provide students with the knowledge and understanding of the mathematics necessary to function in a world very dependent upon the application of mathematics. Instructionally, this goal translates into three components:

- conceptual understanding
- procedural fluency
- problem solving

Conceptual understanding consists of those relationships constructed internally and connected to already existing ideas. It involves the understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts. Students use conceptual understanding of mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts. Knowledge learned with understanding provides a foundation for remembering or reconstructing mathematical facts and methods, for solving new and unfamiliar problems, and for generating new knowledge.

Procedural fluency is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. It includes, but is not limited to, algorithms (the step-by-step routines needed to perform arithmetic operations). Although the word procedural may imply an arithmetic procedure to some, it also refers to being fluent with procedures from other branches of mathematics, such as measuring the size of an angle using a protractor. The use of calculators need not threaten the development of students' computational skills. On the contrary, calculators can enhance both understanding and computing if used properly and effectively. Accuracy and efficiency with procedures are important, but they should be developed through understanding. When students learn procedures through understanding, they are more likely to remember the procedures and less likely to make common computational errors.

Problem solving is the ability to formulate, represent, and solve mathematical problems.

Problems generally fall into three types:

- one-step problems
- multi-step problems
- process problems

Most problems that students will encounter in the real world are multi-step or process problems. Solution of these problems involves the integration of conceptual understanding and procedural knowledge. Students need to have a broad range of strategies upon which to draw. Selection of a strategy for finding the solution to a problem is often the most difficult part of the solution. Therefore, mathematics instruction must include the teaching of many strategies to empower all students to become successful problem solvers. A concept or procedure in itself is not useful in problem solving unless one recognizes when and where to use it as well as when and where it does not apply. Therefore, students need to be able to have a general understanding of how to analyze a problem and how to choose the most useful strategy for solving the problem.

Individually, each of these components (conceptual understanding, procedural fluency, and problem solving) is necessary but not sufficient for a student to be mathematically proficient. They are not, however, independent of each other. They are integrally related, need to be taught simultaneously, and should be a component of every lesson. In this document conceptual understanding, procedural fluency, and problem solving are represented as process strands and content strands. These strands help to define what students should know and be able to do as a result of their engagement in the study of mathematics.

Process Strands: The process strands (Problem Solving, Reasoning and Proof, Communication, and Connections/ Representation) highlight ways of acquiring and using content knowledge. These process strands help to give meaning to mathematics and help students to see mathematics as a discipline rather than a set of isolated skills. Student engagement in mathematical content is accomplished through these process strands. Students will gain a better understanding of mathematics and have longer retention of mathematical knowledge as they solve problems, reason mathematically, prove mathematical relationships, participate in mathematical discourse, make mathematical connections, and model and represent mathematical ideas in a variety of ways.

The Content of Mathematics:

Mathematics is a tool we use to understand and interpret our world. In our increasingly technological economy, those who can understand and apply mathematics have significantly enhanced opportunities to achieve success in continuing education and in life. The key to opening the door to these opportunities is a deep understanding of important mathematical concepts and procedures. The mathematical content must be coherent and vertically articulated across the grades.

The AERO Mathematics Curriculum Framework connects the Process and Content Standards. The Process Standards describe the process in which students should learn mathematics and engage in mathematical thinking. The Content Standards outline the big mathematical ideas that all students should know and be able to do at each grade level. The relationship between the Process and Content Standards is critical. It is the combination of these two standards that will give students mathematical power. Neither will develop mathematically proficient students when used in isolation. Teachers are expected to use instructional practices that provide opportunities for students to experience both Process and Content Standards on a regular basis.

National Council of Teachers of Mathematics (NCTM)

Principles and Standards for School Mathematics

The six **NCTM Principles** describe particular and important features of high-quality mathematics education. The **NCTM Standards** describe the mathematical content and processes that students should learn. Together, the Principles and Standards constitute a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems.

Principles (NCTM)

The Equity Principle

Excellence in mathematics education requires equity – high expectations and strong support for all students. All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study – and support to learn – mathematics. This does not mean that all students should be treated the same. But all students need access each year they are in school to a coherent, challenging mathematics curriculum that is taught by competent and well supported teachers. Equity does not imply lowering expectations for any group of students. Rather expectations must be raised

for all – mathematics can and must be learned by all students. There is no conflict between equity and excellence.

The Curriculum Principle

A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades. School mathematics curricula should focus on mathematics content and processes that are worth the time and attention of students. Mathematics topics can be considered important for different reasons, such as their utility in developing other mathematical ideas, in linking different areas of mathematics, or in deepening students' appreciation of mathematics as a discipline. A coherent curriculum allows students to effectively organize and integrate important mathematical ideas.

The Teaching Principle

Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn. Teachers have different styles and strategies for helping students learn particular mathematical ideas, and there is no one right way” to teach. However, regardless of style, teachers must decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them. Selecting and using suitable curricular material, using appropriate instructional tools and techniques, and engaging in reflective practice and continuous self-improvement are actions good teachers take every day.

A good curriculum is not sufficient for effective learning and teaching. Teachers are required each day to make choices about how the learning environment will be structured and what mathematics will be emphasized. These decisions determine, to a large extent, what students learn. Effective teaching conveys the belief that each student can understand mathematics and that each will be supported in accomplishing this goal.

The Learning Principle

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. Learning with understanding is essential to enable students to solve the new kinds of problems they will inevitably face in the future. Unfortunately, learning without understanding has long been a common outcome of school mathematics. Students who memorize facts and procedures without understanding often are not sure when or how to use what they know, and such learning is often quite fragile. Mathematics makes more sense and is easier to remember and to apply when students connect new knowledge to existing knowledge in meaningful ways. Learning with understanding also makes subsequent learning easier. Students' understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge. Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills.

The Assessment Principle

Assessment should support the learning of important mathematics and furnish useful information to both teachers and students. Assessment should be more than merely a test at the end of instruction to gauge learning. It should be an integral part of instruction that guides teachers and enhances students' learning.

Teachers should be continually gathering information about their students through questions, interviews, writing tasks, and other means. They can then make appropriate decisions about such matters as reviewing materials, re-teaching a difficult concept, or providing something more or different for students who are struggling or need enrichment. To be consistent with the Learning Principle, assessments should focus on understanding as well as procedural skills. Because different students show what they know and can do in different ways, assessments should also be done in multiple ways, such as open-ended questions, constructed-response task, selected-response items, performance tasks, observations, conversations, journals, and portfolios. Constructed-response or performance-tasks may better measure a student's capacity to apply mathematics in complex or new situations.

The Technology Principle

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. Calculators and computers are reshaping the mathematics landscape, and school mathematics should reflect those changes. Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction. Every student should have access to appropriate technology to facilitate his or her mathematics learning. Technology also offers options for students with special needs. Some students may benefit from the more constrained and engaging task situations possible with computers. Students with physical challenges can become much more engaged in mathematics using special technologies. Technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students' mathematical thinking.

NCTM Teacher Edition
NCTM Principles and Standards
Access at www.nctm.org

Process Standards
Standard 1 (Problem Solving),
2(Reasoning and Proof),
3(Communication and Reasoning) , and
4 (Connections)

Enduring Understandings:

Mathematics can be used to solve problems outside of the mathematics classroom.
Mathematics is built on reason and always makes sense.
Reasoning allows us to make conjectures and to prove conjectures.
Classifying helps us build networks of mathematical ideas.
Precise language helps us express mathematical ideas and receive them.

Essential Questions:

Is your plan working?
Do you need to reconsider what you are doing?
How are solving and proving different?
How are showing and explaining different?
How do you know when you have proven something?
How do you develop a convincing argument?
How do you make sense of different strategies?
How do you determine the strengths and weaknesses of different strategies?
How do you determine similarities and differences of different strategies?
Why do we classify numbers?
Why do we classify geometric objects?

1.0 Problem Solving

Problem solving means engaging in a task for which the solution process is not known in advance. Good problem solvers have developed a “mathematical disposition” which allows them to analyze situations in mathematical terms. They have developed a range of strategies for developing a solution to a problem, have learned to monitor and adjust the strategies they choose to use in the process of solving a specific problem, and can compare and contrast solutions and problems.

Students will develop their ability to SOLVE PROBLEMS by engaging in developmentally appropriate problem-solving opportunities in which there is a need to use various approaches to investigate and understand mathematical concepts; to formulate their own problems; to find solutions to problems from everyday situations; to develop and apply strategies to solve a wide variety of problems; and to integrate mathematical reasoning, communication, and connections.. All students in grades K–8 will be able to:

- Develop and apply strategies to solve problems.
- Use mathematical notation and language to explain and defend their thinking
- Make and test conjectures in a variety of mathematical situations.
- Evaluate the reasonableness of the solution in the context of the original situation.

| Standard | 1.0 Students will apply a wide variety of mathematical concepts, processes, and skills to solve a broad range of problems in various content areas and everyday situations. | | | | | | | | | |
|------------------------------------|--|---|---|--|---|---|--|---|---|--|
| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| Indicator | Students will be able to | | | | | | | | | |
| Problem Solving Examine | Identify questions to be answered when solving a problem. | | | Analyze a problem to determine the question(s) to be answered. | | | Extract and organize mathematical information for a given purpose, such as making conjectures or drawing conclusions | | | |
| | Identify what is known and unknown in a problem and recognize when information is missing. | | | Identify necessary and extraneous information | | | Identify necessary and extraneous information | | | |

| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------------------------|---|---|---|---|---|---|---|---|---|
| Indicator | Students will be able to: | | | | | | | | |
| Problem Solving Plan | NA | | | Determine an efficient strategy, verify, interpret, and evaluate the results with respect to the original problem | | | Determine an efficient strategy, verify, interpret, and evaluate the results with respect to the original problem | | |
| Problem Solving Explore | Try more than one strategy when the first strategy proves to be unproductive | | | Try more than one strategy when the first strategy proves to be unproductive | | | Apply problem solving strategies until a solution is found or it is clear that no solution exists | | |
| Problem Solving Solve | Solve problems, choosing from a variety of problem-solving strategies such as drawing pictures, manipulating objects, using numbers, or acting out the situation. | | | Select and use strategies and procedures to find solutions to problems | | | Identify relevant mathematical information in a problem situation and select and use the strategy to solve a problem. | | |
| | NA | | | Interpret and solve a variety of mathematical problems by paraphrasing | | | Interpret and solve a variety of mathematical problems by paraphrasing | | |
| | Check the reasonableness of a solution | | | Check the reasonableness of a solution | | | Check the reasonableness of a solution | | |
| | Explain and verify results with respect to the original problem | | | Generalize and apply previous experiences and strategies to new problem solving situations | | | Generalize solutions and apply previous knowledge to new problem solving situations | | |

| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------------------------|---------------------------|---|---|--|---|---|---|---|---|
| Indicator | Students will be able to: | | | | | | | | |
| Appropriate Technology and Models | NA | | | Use technology, including calculators, to develop mathematical concepts | | | Apply technology as a tool in problem solving situations | | |

2.0 Reasoning and Proof

Systematic reasoning is a defining feature of mathematics. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. Students will develop their Reasoning and **Proof** ability by solving problems in which there is a need to investigate significant mathematical ideas in all content areas; to **justify** their thinking; to reinforce and extend their logical reasoning abilities; to reflect on and clarify their own thinking; to ask questions to extend their thinking; and to construct their own learning. All students in grades K–12 will be able to:

- Reinforce and extend their logical reasoning abilities
- Reflect on, clarify, and justify their thinking
- Ask questions to extend their thinking
- Use patterns and relationships to analyze mathematical situations
- Determine relevant, irrelevant, and/or sufficient information to solve mathematical problems

| Standard | 2.0 Students will apply mathematical reasoning skills to investigate, evaluate, justify, and connect approaches and solutions to situations in mathematics and in other disciplines. | | | | | | | | |
|-----------------------------|---|---|---|--|---|---|--|---|---|
| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Indicator | Students will be able to: | | | | | | | | |
| Reasoning and Proof: | Draw logical conclusions about mathematical problems | | | Draw logical conclusions about mathematical problems | | | Draw logical conclusions about mathematical problems | | |
| | NA | | | Follow a logical argument and judge its validity | | | Recognize and apply deductive and inductive reasoning | | |
| | Discuss the steps used to solve a mathematical problem | | | Review and refine the assumptions and steps used to derive conclusions in mathematical arguments | | | Review and refine the assumptions and steps used to derive conclusions in mathematical arguments | | |
| | Justify and explain the solutions to problems using physical models | | | Justify and explain the solutions to problems using manipulatives and physical models | | | Justify answers and the steps taken to solve problems with and without manipulatives and physical models | | |

3.0 Communication and Representation

As students are asked to communicate orally or in writing about the mathematics they are studying, they gain insights into their own thinking. In order to communicate their thinking to others, they naturally reflect on their learning and organize and consolidate their thinking about mathematics. Students should be encouraged and expected to increase their ability to express themselves clearly and coherently over time. In particular, the ability to express thoughts and describe solutions in writing should be a major focus of the mathematics curriculum.

Representations are necessary to students' understanding of mathematical concepts and relationships. They allow students to communicate mathematical approaches, arguments, and understandings to themselves and others. Appropriate representations allow students to recognize connections among related concepts, and lead to efficient methods of solving problems. It is important to encourage students to represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional. At the same time, students should learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.

Students will develop their mathematical Communication ability by solving problems in which there is a need to obtain information from the real world through reading, listening and observing; to translate this information into mathematical language and symbols; to process this information mathematically; and to present results in written, oral, and visual formats. All students in grades K–12 will be able to:

- Translate information into mathematical language and symbols
- Process information mathematically
- Present results in written, oral, and visual formats
- Discuss and exchange ideas about mathematics as a part of learning
- Read a variety of fiction and nonfiction texts to learn about mathematics
- Use representations to model, communicate and explain problems
- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems

| Standard | 3.0 Students will accurately and clearly present and justify mathematical ideas in diverse formats. | | | | | | | | |
|---------------|--|---|---|--|---|---|--|---|---|
| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Indicator | Students will be able to | | | | | | | | |
| Communication | Use inquiry techniques to solve mathematical problems | | | Use inquiry techniques to solve mathematical problems | | | Use formulas, algorithms, inquiry, and other techniques to solve mathematical problems | | |
| | Use physical materials, models, pictures, or writing to represent and communicate mathematical ideas | | | Use a variety of methods to represent and communicate mathematical ideas through oral, verbal, and written formats | | | Evaluate written and oral presentations in mathematics | | |
| | Identify and translate key words and phrases that imply mathematical operations | | | Identify and translate key words and phrases that imply mathematical operations | | | Identify and translate key words and phrases that imply mathematical operations | | |
| | NA | | | NA | | | Model and explain mathematical relationships using oral, written, graphic, and algebraic methods | | |
| | Explain what they did to solve a problem. | | | Use informal and mathematical language to explain why certain strategies or procedures were used to find a solution. | | | Use appropriate representations, symbols, and informal and formal mathematical language to communicate mathematical thinking coherently and clearly. | | |

4.0 Connections

Mathematics is an integrated field of study, even though it is often studied in separate areas or topics. Viewing mathematics as a whole helps students learn that mathematics is not a set of isolated skills and arbitrary rules. Focusing on mathematics in context and establishing mathematical connections makes it easier to apply mathematical knowledge and makes it less likely that students will forget or misapply important mathematical skills and rules.

Students will develop mathematical Connections by solving problems in which there is a need to view mathematics as an integrated whole and to integrate mathematics with other disciplines, while allowing the flexibility to approach problems, from within and outside mathematics, in a variety of ways. All students in grades K–12 will be able to:

- Link new concepts to prior knowledge
- Identify relationships between content strands
- Allow the flexibility to approach problems in a variety of ways within and beyond the field of mathematics
- Recognize and apply mathematics in contexts outside of mathematics

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|--------------------|---|----------|----------|---|----------|----------|---|----------|----------|
| Standard | 4.0 Students will develop the ability to use connections among mathematical ideas to build on one another when solving real-world problems and to interconnect ideas to produce an integrated coherent whole. | | | | | | | | |
| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Indicators | Students will be able to | | | | | | | | |
| Connections | Apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as rhythm in music and motion in science | | | Use mathematical ideas from one area of mathematics to explain an idea from another area of mathematics | | | Use mathematical ideas from one area of mathematics to explain an idea from another area of mathematics | | |
| | NA | | | Use physical models to explain the relationship between concepts and procedures | | | Use manipulatives and physical models to explain the relationships between concepts and procedures | | |

| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------|--|---|---|---|---|---|---|---|---|
| Indicators | Students will be able to | | | | | | | | |
| Connections | NA | | | NA | | | Use the connections among mathematical topics to develop multiple approaches to problems | | |
| | NA | | | Apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as rhythm in music and motion in science | | | Apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as rhythm in music and motion in science | | |
| | Identify mathematics used in everyday life | | | Identify, explain, and use mathematics in everyday life | | | Identify, explain, and apply mathematics in everyday life | | |

**NCTM Curriculum Focal Points
(Number Sense and Operations)
K**

| Level | Focal Point | Connection to Focal Points |
|----------|---|----------------------------|
| K | <p>Number and Operations: Representing, comparing, and ordering whole numbers and joining and separating sets</p> <p>Children use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set, creating a set with a given number of objects, comparing and ordering sets or numerals by using both cardinal and ordinal meanings, and modeling simple joining and separating situations with objects. They choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the number in a small set, counting and producing sets of given sizes, counting the number in combined sets, and counting backward.</p> | |

**NCTM Curriculum Focal Points
(Number Sense and Operations)
Grade 1**

| Level | Focal Point | Connection to Focal Points |
|----------------|--|--|
| Grade 1 | <p>Number and Operations and Algebra: Developing understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts.</p> <p>Children develop strategies for adding and subtracting whole numbers on the basis of their earlier work with small numbers. They use a variety of models, including discrete objects, length-based models (e.g., lengths of connecting cubes), and number lines, to model “part-whole,” “adding to,” “taking away from,” and “comparing” situations to develop an understanding of the meanings of addition and subtraction and strategies to solve such arithmetic problems. Children understand the connections between counting and the operations of addition and subtraction (e.g., adding two is the same as “counting on” two). They use properties of addition (commutativity and associativity) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.</p> | <p>Number and Operations and Algebra:</p> <p>Children use mathematical reasoning, including ideas such as commutativity and associativity and beginning ideas of tens and ones, to solve two-digit addition and subtraction problems with strategies that they understand and can explain. They solve both routine and nonroutine problems.</p> |

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| | <p>Number and Operations: Developing an understanding of whole number relationships, including grouping in tens and ones.</p> <p>Children compare and order whole numbers (at least to 100) to develop an understanding of and solve problems involving the relative sizes of these numbers. They think of whole numbers between 10 and 100 in terms of groups of tens and ones (especially recognizing the numbers 11 to 19 as 1 group of ten and particular numbers of ones). They understand the sequential order of the counting numbers and their relative magnitudes and represent numbers on a number line.</p> | |
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**NCTM Curriculum Focal Points
(Number Sense and Operations)
Grade 2**

| Level | Focal Point | Connection to Focal Points |
|----------------|---|--|
| Grade 2 | <p>Number and Operations: Developing an understanding of the base-ten numeration system and place-value concepts.</p> <p>Children develop an understanding of the base-ten numeration system and place-value concepts (at least to 1000). Their understanding of base-ten numeration includes ideas of counting in units and multiples of hundreds, tens, and ones, as well as a grasp of number relationships, which they demonstrate in a variety of ways, including comparing and ordering numbers. They understand multidigit numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones).</p> | <p>Number and Operations:</p> <p>Children use place value and properties of operations to create equivalent representations of given numbers (such as 35 represented by 35 ones, 3 tens and 5 ones, or 2 tens and 15 ones) and to write, compare, and order multidigit numbers. They use these ideas to compose and decompose multidigit numbers. Children add and subtract to solve a variety of problems, including applications involving measurement, geometry, and data, as well as nonroutine problems. In preparation for grade 3, they solve problems involving multiplicative situations, developing initial understandings of multiplication as repeated addition</p> |

Number and Operations and Algebra: Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction.

Children use their understanding of addition to develop quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition (commutativity and associativity). Children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multidigit whole numbers. They select and apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including standard algorithms, for adding and subtracting whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.

**NCTM Curriculum Focal Points
(Number Sense and Operations)
Grade 3**

| Level | Focal Point | Connection to Focal Points |
|----------------|---|---|
| Grade 3 | <p>Number and Operations and Algebra: Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts.</p> <p>Students understand the meanings of multiplication and division of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal “jumps” on number lines for multiplication, and successive subtraction, partitioning, and sharing for division). They use properties of addition and multiplication (e.g., commutativity, associativity, and the distributive property) to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts. By comparing a variety of solution strategies, students relate multiplication and division as inverse operations.</p> | <p>Number and Operations:</p> <p>Building on their work in grade 2, students extend their understanding of place value to numbers up to 10,000 in various contexts. Students also apply this understanding to the task of representing numbers in different equivalent forms (e.g., expanded notation). They develop their understanding of numbers by building their facility with mental computation (addition and subtraction in special cases, such as $2,500 + 6,000$ and $9,000 - 5,000$), by using computational estimation, and by performing paper-and-pencil computations.</p> |

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| | <p>Number and Operations: Developing an understanding of fractions and fraction equivalence.</p> <p>Students develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models, including the number line, to identify equivalent fractions.</p> | |
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**NCTM Curriculum Focal Points
(Number Sense and Operations)
Grade 4**

| Level | Focal Point | Connection to Focal Points |
|----------------|---|--|
| Grade 4 | <p>Number and Operations and Algebra: Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication.</p> <p>Students use understandings of multiplication to develop quick recall of the basic multiplication facts and related division facts. They apply their understanding of models for multiplication (i.e., equal sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multidigit whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for multiplying whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.</p> | <p>Number and Operations:</p> <p>Building on their work in grade 3, students extend their understanding of place value and ways of representing numbers to 100,000 in various contexts. They use estimation in determining the relative sizes of amounts or distances. Students develop understandings of strategies for multidigit division by using models that represent division as the inverse of multiplication, as partitioning, or as successive subtraction. By working with decimals, students extend their ability to recognize equivalent fractions. Students' earlier work in grade 3 with models of fractions and multiplication and division facts supports their understanding of techniques for generating equivalent fractions and simplifying fractions.</p> |

Number and Operations: Developing an understanding of decimals, including the connections between fractions and decimals.

Students understand decimal notation as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They connect equivalent fractions and decimals by comparing models to symbols and locating equivalent symbols on the number line.

**NCTM Curriculum Focal Points
(Number Sense and Operations)
Grade 5**

| Level | Focal Point | Connection to Focal Points |
|----------------|--|--|
| Grade 5 | <p>Number and Operations and Algebra: Developing an understanding of and fluency with division of whole numbers.</p> <p>Students apply their understanding of models for division, place value, properties, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multidigit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for dividing whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems. They consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately.</p> | <p>Number and Operations: Building on their work in grade 4, students extend their understanding of place value to numbers through millions and millionths in various contexts. They apply what they know about multiplication of whole numbers to larger numbers. Students also explore contexts that they can describe with negative numbers (e.g., situations of owing money or measuring elevations above and below sea level).</p> |

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| | <p>Number and Operations: Developing an understanding of and fluency with addition and subtraction of fractions and decimals.</p> <p>Students apply their understandings of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They apply their understandings of decimal models, place value, and properties to add and subtract decimals. They develop fluency with standard procedures for adding and subtracting fractions and decimals. They make reasonable estimates of fraction and decimal sums and differences. Students add and subtract fractions and decimals to solve problems, including problems involving measurement.</p> | |
| | | |

**NCTM Curriculum Focal Points
(Number Sense and Operations)
Grade 6**

| Level | Focal Point | Connection to Focal Points |
|----------------|--|--|
| Grade 6 | <p>Number and Operations: Developing an understanding of and fluency with multiplication and division of fractions and decimals.</p> <p>Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.</p> | <p>Number and Operations:</p> <p>Students' work in dividing fractions shows them that they can express the result of dividing two whole numbers as a fraction (viewed as parts of a whole). Students then extend their work in grade 5 with division of whole numbers to give mixed number and decimal solutions to division problems with whole numbers. They recognize that ratio tables not only derive from rows in the multiplication table but also connect with equivalent fractions. Students distinguish multiplicative comparisons from additive comparisons.</p> |

Number and Operations: Connecting ratio and rate to multiplication and division.

Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., “If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12”). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.

**NCTM Curriculum Focal Points
(Number Sense and Operations)
Grade 7**

| Level | Focal Point | Connection to Focal Points |
|----------------|--|--|
| Grade 7 | <p>Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity.</p> <p>Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ($y/x = k$, or $y = kx$) from other relationships, including inverse proportionality ($xy = k$, or $y = k/x$).</p> | <p>Number and Operations:</p> <p>In grade 4, students used equivalent fractions to determine the decimal representations of fractions that they could represent with terminating decimals. Students now use division to express any fraction as a decimal, including fractions that they must represent with infinite decimals. They find this method useful when working with proportions, especially those involving percents. Students connect their work with dividing fractions to solving equations of the form $ax = b$, where a and b are fractions. Students continue to develop their understanding of multiplication and division and the structure of numbers by determining if a counting number greater than 1 is a prime, and if it is not, by factoring it into a product of primes.</p> |

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| | | |
| | <p>Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations.</p> <p>Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts (e.g., situations of owing money or measuring elevations above and below sea level), students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. They use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.</p> | |

**NCTM Curriculum Focal Points
(Number Sense and Operations)
Grade 8**

| Level | Focal Point | Connection to Focal Points |
|---------|---|---|
| Grade 8 | <p>Data Analysis and Number and Operations and Algebra: Analyzing and summarizing data sets.</p> <p>Students use descriptive statistics, including mean, median, and range, to summarize and compare data sets, and they organize and display data to pose and answer questions. They compare the information provided by the mean and the median and investigate the different effects that changes in data values have on these measures of center. They understand that a measure of center alone does not thoroughly describe a data set because very different data sets can share the same measure of center. Students select the mean or the median as the appropriate measure of center for a given purpose.</p> | <p>Number and Operations: Students use exponents and scientific notation to describe very large and very small numbers. They use square roots when they apply the Pythagorean theorem.</p> |

Grade 8

See Data Analysis Standard

5.0 NUMBERS AND OPERATIONS

Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts. Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Enduring Understandings:

Numbers can be represented in multiple ways.

The same operations can be applied in problem situations that seem quite different from another.

Being able to compute fluently means making smart choices about which tools to use and when to use them.

Knowing the reasonableness of an answer comes from using good number sense and estimation strategies.

Essential Questions:

1. What makes an estimate reasonable?
2. What makes an answer exact?
3. What makes a strategy both effective and efficient?
4. What makes a solution optimal?

**5.0 NUMBERS and OPERATIONS
STANDARDS and PERFORMANCE INDICATORS**

| | | | | | | | | | |
|-------------------------|--|---|--|--|--|--|----------|----------|----------|
| Standard | 5.0. Students will understand and apply numbers, ways of representing numbers, relationships among numbers, and number systems. | | | | | | | | |
| Benchmark | 5.1 Numbers and Number Sense Students will understand and demonstrate a sense of what numbers mean and how they are used. Students will be able to: | | | | | | | | |
| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Counting | Count forward by 1's to 31 and backward from 10 with and without objects | Count forward by 1's to 99 and backward from 20 with and without objects and count by two's to at least 100 | Count by tens or hundreds forward and backward starting at any number from 1 to 999 and count by fives and tens to at least 100. | Count by hundreds and thousands starting at any number from 1 to 9,999 | Count by thousands and ten thousands starting at any number from 1 to 99,999 | Count by thousands, ten thousands, and hundred thousands, starting at any number from 1 to 999,999 | NA | NA | NA |

| Level Indicators | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|-----------------------------|--|---|--|--|--|--|----|----|----|
| | Counting Sets | Count how many objects are in a set of up to 10 objects and count out a specific number of objects (up to 10) from a larger set. | Group and count objects by tens, fives, and twos to 100 | Name the number that is 1 more than or 10 more than any number from 0 through 999 and 1 less than any number from 10 through 1000. | Name the number that is 10 more than or 100 more than any number from 0 through 9,999 and 10 less than any number from 100 through 10,000. | Name the number that is 100 more than or 1000 more than any number from 0 through 99,999 and 100 less than any number from 1000 through 100,000. | NA | NA | NA | NA |
| | Reading and Writing Numbers | Identify and read aloud numbers from 0 to at least 31 | Identify, read aloud and write numbers to 100 | Identify, read aloud and write numbers to 1000. | Identify, read aloud and write numbers to 10,000. | Read and write numbers to at least 100,000 | Read and write numbers to at least 1,000,000 | NA | NA | NA |

| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|--|--|--|---|--|--|----|----|----|
| Ordering and Comparing (Whole Numbers) | Compare two sets of up to 10 objects each and explain why the number of objects in one set is equal to, greater than, or less than the number of objects in the other set. | Write, compare, and order numbers to at least 100 using the words <i>equal to</i> , <i>greater than</i> , <i>less than</i> , <i>greatest</i> , and <i>least</i> when appropriate . | Compare and order numbers from 0 to at least 1,000 using the words <i>equal to</i> , <i>greater than</i> , <i>less than</i> , <i>greatest</i> , or <i>least</i> when appropriate . | Compare and order numbers from 0 to at least 10,000 using the words <i>equal to</i> , <i>greater than</i> , <i>less than</i> , <i>greatest</i> , or <i>least</i> when appropriate | Compare and order numbers from 0 to at 100,000 using the words <i>equal to</i> , <i>greater than</i> , <i>less than</i> , <i>greatest</i> , or <i>least</i> when appropriate | Compare and order numbers from 0 to at 1,000,000 using the words <i>equal to</i> , <i>greater than</i> , <i>less than</i> , <i>greatest</i> , or <i>least</i> when appropriate | NA | NA | NA |

| LEVEL Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------------------------------------|---|--|---|--|---|---|---|---|---|
| Ordering and Comparing (Numbers) | Use a number line or chart to locate and identify the numbers (from 1 to 31) coming before/after a given number and between 2 given numbers | Use a number line or chart, locate, compare, and order whole numbers less than 100 and identify the numbers coming before/after a given number and between 2 given numbers | Use words, number lines, and models to compare, and order whole numbers through 999 | Use symbols (i.e., $<$, $=$, $>$) and models to compare and order whole numbers through 9,999 | Use symbols (i.e., $<$, $=$, $>$) and models to compare and order whole numbers through 99,999 | Read, write, compare, and order all whole numbers, fractions, mixed numbers and decimals using multiple strategies (e.g. symbols, manipulatives, number line, and place value concepts) | Read, write, compare, and order integers using multiple strategies (e.g., symbols, manipulatives, number line). | Compare, order, and differentiate among integers, decimals, fractions, and irrational numbers using multiple representations (e.g., symbols, manipulatives, graphing on a number line). | Analyze, describe and compare the characteristics of rational and irrational numbers. |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|---|--|---|---|---|--|---|--|---|
| Place Value | Identify place value of each digit utilizing standard and expanded form through 20. | Construct models and identify place value of each digit utilizing standard and expanded form through 99. | Construct models and identify place value of each digit utilizing standard and expanded form through 999. | Model and identify place value of each digit utilizing standard and expanded form through 9999. | Identify and interpret the place value for each digit in numbers through 99,999 | Identify and use place value positions of whole numbers and decimals to hundredths | Identify and use place value positions of whole numbers and decimals to thousandths | Write, identify, and use (standard and expanded form) powers of 10 from 10^{-3} through 10^6 | Represent numbers using scientific notation in mathematical and practical situations. |
| Exponents | NA | NA | NA | NA | NA | NA | NA | Explain the relationship between standard form and scientific notation. | Recognize and appropriately use exponential and scientific notation. |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|----|----|----|----|----|----|
| Ordering and Comparing (Ordinal Numbers) | Use the ordinal numerals 1 st through 10 th to discuss positions in ordered lists | Use ordinal numbers 1 st – 20 th to identify position in a sequence | Use ordinal numbers through 31 st as they relate to the calendar | NA | NA | NA | NA | NA | NA |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------------------------------|---|--|---|---|--|---|---|---|---|
| Fractions and Decimals | Identify and name halves and whole using concrete items | Identify and name halves, thirds, and fourths as part of a whole and as part of a group using models | Represent fractions that have denominators ranging from 2 to 12 using physical objects, pictures, numbers, and words, and translate among representations | Use concrete models and pictorial representations to demonstrate the meaning of fractions (proper and improper) as parts of a whole, parts of a set, and division by whole numbers through twelfths | Use models to connect and compare equivalent fractions and decimals. | Determine decimal equivalents or approximations of common fractions (i.e., $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 whole) | Determine decimal and percent equivalents including approximations for common fractions (i.e., $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 whole) , | Represent rational numbers as fractions, mixed numbers, decimals or percents and convert among various forms as appropriate | Classify numbers as rational or irrational. |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|----------------------|----|----|--|---|--|---|--|---|--|
| Equivalent Fractions | NA | NA | Identify that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one. | Identify, name and use equivalent fractions with denominators 2, 4 and 8. | Write a fraction equivalent to a given fraction using common multiples. And simplify fractions using common factors. | Relate equivalent fractions and decimals with and without models, including locations on a number line. | Compute equivalent representations of fractions and decimals (i.e., halves, thirds, fourths, fifths, eighths, tenths, hundredths) | Model and identify equivalent fractions including conversion of improper fractions to mixed numbers and vice versa. | Use rational and irrational numbers to solve real-world and mathematical problems. |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|----|----|--|---|--|---|--|-----------------------------------|--|
| <p>Comparing Decimals</p> <p>NOT FOR PUBLICATION AERO MATHEMATICS © AERO 2009</p> | NA | NA | Place 0 and halves on the number line from 0 to 10 | Compare and order fractions by using models, benchmarks (0, $\frac{1}{2}$, 1), or common numerators or denominators. | Compare and order positive fractions (including positive mixed numbers) and decimals on the number line, in number sentences, and in lists | Use models and drawings, and find common denominators to compare fractions with unlike denominators | Compare positive fractions, decimals, and positive and negative integers using symbols (i.e., $<$, $=$, $>$) and number lines | Compare and order combinations of | Represent and compare rational and irrational numbers symbolically and on a number line. |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------------------|----|----|----|---|---|---|--|----|----|
| Representing Decimals | NA | NA | NA | Use numbers, words, pictures, and physical objects to read, write, and represent decimal numbers (to the tenths) between 0 and 1, between 1 and 2, etc. | Use numbers, words, pictures, and physical objects to read, write, and represent decimal numbers (to the hundredths) between 0 and 1, between 1 and 2, etc. | Round, order, and compare, using symbols, decimals to the tenths, hundredths, and thousandths place | Read, write, compare, and order groups of decimals | NA | NA |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|--|--|--|--|---|---|---|---|
| Equivalency Decimals and Fractions | NA | NA | Distinguish the equivalency among decimals, fractions and percents (e.g., half = 50%). | Determine the equivalency among decimals, fractions, and percents (e.g., half = 50% and $\frac{1}{4} = 0.25 = 25\%$). | Determine the equivalency among decimals, fractions, and percents (e.g., $\frac{49}{100} = 0.49 = 49\%$). | Determine the equivalency between and among fractions, decimals, and percents in contextual situations. | Determine the equivalency between and among fractions, decimals, and percents in contextual situations. | Express fractions as terminating or repeating decimals. | NA |
| Counting Money | Identify and sort coins of the host country | Find the value of any set of coins using one denomination of coins | Determine the value of a given set of coins | Determine possible combinations of coins and bills to equal given amounts | Determine totals for monetary amounts in practical situations | Determine totals, differences, and change due for monetary amounts in practical situations | Compare and use unit cost in practical situations | Calculate simple interest in monetary problems | Calculate percents in monetary problems |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|----|----|------------------------------------|------------------------------------|---|----|----|----|----|
| Money Notation | NA | NA | Use decimals to show money amounts | Read, write and use money notation | Use money notation to add and subtract given monetary amounts | NA | NA | NA | NA |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|----|----|----|---|---|--|--|---|---|
| Integers | NA | NA | NA | NA | Illustrate the meaning of positive and negative integers using models, such as the number line or colored chips, and situations, such as elevation or temperature | Describe real-world situations using positive and negative numbers | Explain the meaning and relationship between absolute value and opposites. | Develop, analyze, and apply models (including everyday contexts), strategies, and procedures to compute with integers, with an emphasis on negative integers. | Simplify expressions involving operations on integers, grouping symbols, and whole number exponents using order of operations |
| Ratios | NA | NA | NA | Describe relationships between quantities using ratios. | Make comparisons and describe quantitative relationships using ratios. | Represent ratios and proportions and solve problems using models and pictures. | Compare quantities and solve problems using ratios, rates and percents. | Use percents to make comparisons between groups of unequal size. | Apply ratio and proportionality to solve problems, including percent and simple probability |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|----|----|----|----|----|----|--|---|---|
| Proportions | NA | NA | NA | NA | NA | NA | Write and apply ratios in mathematical and practical situations involving measurement and monetary conversions | Write and apply proportions to solve mathematical and practical problems involving measurement and monetary conversions | Apply ratios and proportions to calculate rates and solve mathematical and practical problems using indirect measure. |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|----|----|----|----|----|--|---|--|---|
| Percentages | NA | NA | NA | NA | NA | Identify and represent ratios as comparisons of part-to-part and part-to-whole relationships, and solve problems involving ratios. | Represent percents in various forms using numbers, pictures, models, or circle graphs and solve problems involving percentages. Greater than 100 and less than 1. | Calculate the percentage of increase and decrease of a quantity in real-world and mathematical problems. | Solve contextual problems using ratios, rates, or percents and verify the reasonableness of the solution. |

| Benchmark | 5.2 Operations on Numbers Students will understand meanings of operations and how they relate to one another. Students will be able to: | | | | | | | | |
|---|--|--|---|--|--|---|---|----|----|
| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Computation Whole numbers | Use concrete objects to model simple joining and separating situations (addition and subtraction) of whole numbers related to sums of 10 or less and write corresponding number sentence. | Use concrete objects to model the addition of two or three addends and subtraction of whole numbers related to sums less than 20 and write the corresponding number sentence | Demonstrate efficient procedures for adding and subtracting 2 and 3 digit whole numbers and explain why the procedures work on the basis of place value and number properties | Apply models of multiplication (e.g., equal-sized groups, arrays, area models, equal “jumps” on number lines and hundreds charts) and division (e.g., repeated subtraction, partitioning, and sharing) to solve problems | Add and subtract whole numbers (up to five – digit number) | Add, subtract, multiply, and divide (with and without remainders) using non-negative rational numbers. | Model addition and subtraction of integers with physical materials and the number line. | NA | NA |

| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------------------|-----|--|---|---|--|--|--|----|----|
| Meaning of Operations | NA. | Use movement on the number line to demonstrate the inverse relationship between addition and subtraction | Model, represent, and explain multiplication (products to 81) as a rectangular array, as repeated addition and skip counting, or as equal-sized moves on the number line and division as repeated subtraction, sharing and grouping | Apply the inverse relationship between multiplication and division (e.g., $5 \times 6 = 30$, $30 \div 6 = 5$) and the relationship between multiples and factors. | Represent multiplication of two-digit by two-digit numbers and describe how that representation connects to the related number sentence. | Multiply four-digit numbers by two-digit numbers (including whole numbers and decimals). | Use various methods to find quotients for multi-digit division problems. And justify why the procedures work on the basis of place value and number properties | NA | NA |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------------------|----|----|----|--|---|--|--|----|----|
| Operations with fractions | NA | NA | NA | Use concrete models to add and subtract simple common fractions with the same denominator. | Use decimal models, place value, and number properties to add and subtract decimals (to the thousandths). | Model addition and subtraction of mixed numbers with and without regrouping and fractions with like and unlike denominators. | Apply the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and justify why they work. | NA | NA |

| Level | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|-----------------------------|----|----|----|--|--|---|---|---|--|
| | Prime and composite Numbers | NA | NA | NA | Identify whole number factors and/or pairs of factors for a given whole number through 24. | Identify factors of composite numbers less than 50 | Identify all whole number factors and pairs of factors for a given whole number through 144 | Identify the greatest common factor for a set of whole numbers. | Determine the Greatest Common Factor (GCF) and Least Common Multiple (LCM) of two numbers in the context of problem-solving | Apply the concepts of Greatest Common Factor (GCF) and Least Common Multiple (LCM) to monomials with variables |

| Level Indicators | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|--|----|----|----|--|---|--|---|---|----|
| Numbers | | NA | NA | NA | Illustrate with manipulatives when a number is divisible by 2, 3, 5, or 10 | Use divisibility concepts to classify numbers as prime or composite | Model and distinguish between factor and multiple and prime and composite numbers. | Express a whole number as a product of its prime factors, using exponents when appropriate. | Use the concepts of number theory, including prime and composite numbers, factors, multiples, and the rules of divisibility to solve problems | NA |
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| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|----------------------|----|--|---|--|----|----|----|----|----|
| Odd and Even Numbers | NA | Identify odd and even numbers to 20 and determine if a set of objects has an odd or even number of elements. | Demonstrate the relationships between odd and even numbers in addition and subtraction such as, odd + odd = even or odd – even = odd. | Sort whole numbers into sets containing only odd numbers or only even numbers. | NA | NA | NA | NA | NA |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------|----|---|---|--|--|---|--|----|----|
| Mental Math | NA | Recall from memory single digit addition facts (to 9 + 9) and the corresponding subtraction facts | Carry out addition and subtraction mentally involving: 3-digit numbers and ones; 3-digit numbers and tens; 3-digit numbers and hundreds | Add or subtract with numbers less than 100 using mental arithmetic | Recall from memory multiplication facts for numbers from 1 to 10 | Recall from memory multiplying and dividing by 10, 100, and 1,000 | Develop and use strategies for mental computations with non-negative whole numbers, fractions, and decimals. | NA | NA |

| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------------------|---|---|--|---|--|----|----|----|----|
| Properties of Numbers | Model meanings of operations and the relationship between addition and subtraction (e.g., identity element of addition, commutative property) using manipulatives | Use the concept of commutative $[4 + 2 = 2 + 4]$, associative $[(4 + 3) + 7 = 4 + (3 + 7)]$, and identity $[0 + 3 = 3]$ properties of addition to solve problems involving basic facts. | Model and justify the relationship between addition and subtraction (e.g., identity element of addition, associative property, commutative property, inverse operations, fact families). | Use and explain the operations of multiplication and division including the properties (e.g., identity element of multiplication, commutative property, property of zero, associative property, inverse operations) | Apply models for multiplication (e.g., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (commutative, associative, and distributive). | NA | NA | NA | NA |

| | | | | | | | | | | |
|-----------------|-------------------|---|--|---|---|--|---|---|--|--|
| Benchmark | | 5.3 Numerical Operations and Estimation Students will accurately calculate and use estimation techniques, number relationships, operation rules, and algorithms; they will determine the reasonableness of answers and the accuracy of solutions. | | | | | | | | |
| Level Indicator | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | Estimation | Estimate the number of objects in a group of 20 or less and count to evaluate reasonableness of estimation. | Estimate the number of objects in a group of 100 or less and count to evaluate reasonableness of estimate. | Use rounding to analyze the reasonableness of a sum or a difference.. | Apply estimation skills (rounding, benchmark s, compatible numbers) to solve and evaluate reasonableness of an answer | Estimate solutions to problems including rounding, benchmark s, compatible numbers and evaluate the reasonableness of the solution, justify results. | Use mental math and estimation strategies to predict the results of computations (i.e., whole numbers, addition and subtraction of fractions) and to test the reasonableness of solutions | Use and explain estimation strategies to predict computational results with positive fractions and decimals | Determine the reasonableness of answers involving positive fractions and decimals by comparing them to estimates | Estimate the answer to an operation involving rational numbers based on the original numbers |

| | Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--|------------------|----|---|---|--|---|----|----|----|----|
| | Rounding | NA | Given a number and number line/hundreds chart, identify the nearest ten | Round numbers to the nearest 10 or 100 and identify situations in which rounding is appropriate | Round whole numbers through 10,000 to the nearest ten, hundred, and thousand and round fractions to the nearest whole number | Round whole numbers to 1,000,000 to any place value and round decimals to the nearest whole, 10 th , or 100 th place. | NA | NA | NA | NA |

| Level Indicators | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|-----------------|---|---|---|--|---|---|---|---|---|
| | Problem Solving | Create grade – appropriate story picture and story problems, solve using a variety of strategies, present and justify results | Solve and create a story problem that matches an addition or subtraction expression or equation using physical objects, pictures, or words. | Generate and solve one step addition and subtraction problems based on practical situations | Generate and solve two step addition and subtraction problems and one step multiplication problems based on practical situations | Generate and solve addition, subtraction, multiplication, and division problems using whole numbers in practical situations | Select, sequence, and use appropriate operations to solve multi-step word problems with whole numbers | Solve problems involving addition, subtraction, multiplication, and division of rational numbers and express answers in simplest form | Use the order of operations to simplify and/or evaluate whole numbers (including exponents and grouping symbols). | Simplify and evaluate expressions using order of operations and use real number properties to justify solutions |

| Level Indicators | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|------------------------|----|--|--|---|---|---|--|--|---|
| | Problem Solving | NA | Apply strategies, including counting on, counting back, and doubling, for addition facts to at least 10. | Apply strategies, including counting on, counting back, doubling, and halving, for addition and subtraction facts. | Describe and show relationships between strategies and procedures for multiplying and dividing that involve addition and subtraction and explain strategies | Select and test algorithms used in computational situations that involve multiplication and division of whole numbers and explain strategies. | Select and/or use an appropriate operation(s) to show understanding of addition and subtraction of non-negative decimals and/or fractions | Decide which representation (i.e., fraction or decimal) of a positive number is appropriate in a real-life situation | Set up and solve simple percent problems using various strategies, including mental math | Identify missing information or suggest a strategy for solving a real-life, rational-number problem |

**NCTM Curriculum Focal Points
(Measurement)**

K

| Level | Focal Points | Connection to the Focal Points |
|----------|---|--------------------------------|
| K | <p>Measurement: Ordering objects by measurable attributes</p> <p>Children use measurable attributes, such as length or weight, to solve problems by comparing and ordering objects. They compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object), and they order several objects according to length</p> | |

**NCTM Curriculum Focal Points
(Measurement)
Grade 1**

| Level | Focal Points | Connection to the Focal Points |
|----------------|--------------|---|
| Grade 1 | | <p>Measurement and Data Analysis:</p> <p>Children strengthen their sense of number by solving problems involving measurements and data. Measuring by laying multiple copies of a unit end to end and then counting the units by using groups of tens and ones supports children's understanding of number lines and number relationships. Representing measurements and discrete data in picture and bar graphs involves counting and comparisons that provide another meaningful connection to number relationships</p> |
| | | |
| | | |

**NCTM Curriculum Focal Points
(Measurement)
Grade 2**

| Level | Focal Points | Connection to the Focal Points |
|----------------|---|---|
| Grade 2 | <p>Measurement: Developing an understanding of linear measurement and facility in measuring lengths.</p> <p>Children develop an understanding of the meaning and processes of measurement, including such underlying concepts as partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand linear measure as an iteration of units and use rulers and other measurement tools with that understanding. They understand the need for equal-length units, the use of standard units of measure (centimeter and inch), and the inverse relationship between the size of a unit and the number of units used in a particular measurement (i.e., children recognize that the smaller the unit, the more iterations they need to cover a given length</p> | <p>Geometry and Measurement:</p> <p>Children estimate, measure, and compute lengths as they solve problems involving data, space, and movement through space. By composing and decomposing two-dimensional shapes (intentionally substituting arrangements of smaller shapes for larger shapes or substituting larger shapes for many smaller shapes), they use geometric knowledge and spatial reasoning to develop foundations for understanding area, fractions, and proportions.</p> |

**NCTM Curriculum Focal Points
(Measurement)
Grade 3**

| Level | Focal Points | Connection to the Focal Points |
|----------------|--------------|--|
| Grade 3 | | <p>Measurement:</p> <p>Students in grade 3 strengthen their understanding of fractions as they confront problems in linear measurement that call for more precision than the whole unit allowed them in their work in grade 2. They develop their facility in measuring with fractional parts of linear units. Students develop measurement concepts and skills through experiences in analyzing attributes and properties of two dimensional objects. They form an understanding of perimeter as a measurable attribute and select appropriate units, strategies, and tools to solve problems involving perimeter.</p> |

**NCTM Curriculum Focal Points
(Measurement)
Grade 4**

| Level | Focal Points | Connection to the Focal Points |
|----------------|--|---|
| Grade 4 | <p>Measurement: Developing an understanding of area and determining the areas of two-dimensional shapes.</p> <p>Students recognize area as an attribute of two-dimensional regions. They learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a square that is 1 unit on a side is the standard unit for measuring area. They select appropriate units, strategies (e.g., decomposing shapes), and tools for solving problems that involve estimating or measuring area. Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle.</p> | <p>Measurement: As part of understanding two-dimensional shapes, students measure and classify angles.</p> |

**NCTM Curriculum Focal Points
(Measurement)
Grade 5**

| Level | Focal Points | Connection to the Focal Points |
|----------------|--|--|
| Grade 5 | <p>Geometry and Measurement and Algebra: Describing three-dimensional shapes and analyzing their properties, including volume and surface area.</p> <p>Students relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. They understand that a cube that is 1 unit on an edge is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of prisms. As they work with surface area, they find and justify relationships among the formulas for the areas of different polygons. They measure necessary attributes of shapes to use area formulas to solve problems.</p> | <p>Measurement:</p> <p>Students' experiences connect their work with solids and volume to their earlier work with capacity and weight or mass. They solve problems that require attention to both approximation and precision of measurement.</p> |

**NCTM Curriculum Focal Points
(Measurement)
Grade 6**

| Level | Focal Points | Connection to the Focal Points |
|----------------|---|--------------------------------|
| Grade 6 | <p>Measurement and Geometry:</p> <p>Problems that involve areas and volumes, calling on students to find areas or volumes from lengths or to find lengths from volumes or areas and lengths, are especially appropriate. These problems extend the students' work in grade 5 on area and volume and provide a context for applying new work with equations</p> | |

**NCTM Curriculum Focal Points
(Measurement)
Grade 7**

| Level | Focal Points | Connection to the Focal Points |
|----------------|--|--|
| Grade 7 | <p>Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes.</p> <p>By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders by slicing them, they develop and understand formulas for their volumes ($\text{Volume} = \text{Area of base} \times \text{Height}$). They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students see that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram. They select appropriate two- and three dimensional shapes to model real-world situations and solve a variety of problems (including multistep problems) involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders.</p> | <p>Measurement and Geometry:</p> <p>Students connect their work on proportionality with their work on area and volume by investigating similar objects. They understand that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related. Students apply their work on proportionality to measurement in different contexts, including converting among different units of measurement to solve problems involving rates such as motion at a constant speed. They also apply proportionality when they work with the circumference, radius, and diameter of a circle; when they find the area of a sector of a circle; and when they make scale drawings.</p> |

**NCTM Curriculum Focal Points
(Measurement)
Grade 8**

| Level | Focal Points | Connection to the Focal Points |
|----------------|--|--------------------------------|
| Grade 8 | <p>Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angle.</p> <p>Students use fundamental facts about distance and angles to describe and analyze figures and situations in two- and three-dimensional space and to solve problems, including those with multiple steps. They prove that particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lines. Students apply this reasoning about similar triangles to solve a variety of problems, including those that ask them to find heights and distances. They use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degrees, and they apply this fact about triangles to find unknown measures of angles. Students explain why the Pythagorean theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane to measure lengths and analyze polygons and polyhedra.</p> | |

6.0 MEASUREMENT

Measurement provides a way to answer questions about “how many,” “how much” and “how far.” It is an indispensable component of business, manufacturing, art, medicine and many other aspects of daily life. We describe the sizes, capacities and values of many things, from the large distances involved in space travel, to the very small quantities in computer design and microbiology, to the varying values of currencies in international monetary exchange. All people must be able to choose an appropriate level of accuracy for a measurement; to select what measuring instruments to use and to correctly determine the measures of objects, space and time. These activities require people to be able to use standard instruments including rulers, volume and capacity measures, timers and emerging measurement technologies found in the home and workplace.

ENDURING UNDERSTANDINGS:

Linear measure, area, and volume are fundamentally different but may be related to one another in ways that permit calculation of one given the other.

ESSENTIAL QUESTIONS:

1. How are measurement and counting related?
2. How does *what* we measure affect *how* we measure?
3. How can space be defined through numbers/measurement?

**6.0 MEASUREMENT
STANDARDS AND PERFORMANCE INDICATORS**

| | | | | | | | | | |
|------------------------|---|---|---|---|----------|----------|----------|----------|----------|
| Standard | 6.0 MEASUREMENT Students will use concepts and tools of measurement to describe and quantify the world. Students will be able to: | | | | | | | | |
| Benchmark | 6.1 Physical Attributes Students will demonstrate an understanding of units of measure and measurable attributes of objects., Student will be able to: | | | | | | | | |
| Level Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Calendar | Name in order the days of the week | Name in order the months of the year and use the calendar to identify days, weeks, months, and a year | Recognize that there are 12 months in 1 year, 7 days in 1 week, and 24 hours in 1 day | Recognize the number of weeks in a year, days in a year, and days in each month | NA | NA | NA | NA | NA |

| LEVEL Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------------|---|---|--|--|--|--|----|----|----|
| Elapsed Time | Sequence events; and identify calendars and clocks as objects that measure time | Sequence events with respect to time; e.g., yesterday, today, tomorrow, seasons | Use elapsed time in one hour increments , beginning on the hour, to determine start, end, and elapsed time | Use elapsed time in half-hour increments , beginning on the hour or half-hour, to determine start, end, and elapsed time | Use elapsed time in quarter-hour increments , beginning on the quarter-hour, to determine start, end, and elapsed time | NA | NA | NA | NA |
| Time | Tell time to the hour using digital and analog clocks | Tell time to the hour and half-hour using digital and analog clocks | Tell time to the nearest quarter hour and 5 minute interval using digital and analog clocks | Tell time to the nearest minute using digital and analog clocks | Use A.M. and P.M. appropriately in describing time | Determine equivalent periods of time, including relationships between and among seconds, minutes, hours, days, months, and years | NA | NA | NA |

| Level Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------------------------------|--|--|---|--|--|--|---|---|---|
| Comparison, Estimation and Conversion | Use comparative vocabulary in measurement settings (e.g., long/longer, short/shorter, more/less, hotter/colder, heavier/lighter, bigger/smaller) | Compare, order, describe, and represent objects by length and weight | Compare, order, and describe objects by various measurable attributes for length, weight, and temperature | Compare, order, and describe objects by various measurable attributes for area and volume/capacity | Estimate and convert units of measure for length, area, and weight with the same measurement system (metric) Estimate temperature in practical situations | Estimate and convert units of measure for weight and volume/capacity within the same measurement system (metric) | Estimate and compare units of measure for temperature, length, and weight/mass metric systems | Estimate and compare corresponding units of measure for area and volume/capacity metric systems | Estimate and convert units of measure for mass and capacity within the same measurement system (metric) |

| Benchmark | 6.2 Systems of Measurement Students will identify and use units, systems and processes of measurement. Students will be able to: | | | | | | | | |
|-----------|---|---|--|--|---|--|--|--|--|
| LEVEL | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Measuring | Identify measurable attributes, such as length, weight, and capacity, and use these attributes to make direct comparisons. | Estimate and verify by measuring, length, weight, and capacity using nonstandard units (e.g., sticks, paper clips, blocks, beans) | Select and use appropriate tools and units to measure length, time, capacity, and weight (e.g., scales for kilograms; rulers for centimeters; measuring containers for cup, and liters) thermometer in degrees Celsius | Select and use appropriate units of measure and measure to a required degree of accuracy (to the nearest $\frac{1}{2}$ unit) | Measure length, area, weight, and temperature, to a required degree of accuracy in metric systems | Measure volume and weight to a required degree of accuracy in the metric systems | Explain how the size of the unit of measure used affects precision | Estimate a measurement to the degree of precision that the tool provides | Select an appropriate degree of precision when using measurements for calculations |

| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------|--|--|--|---|--|---|--|--|----|
| Units | Measure length with non-standard units; e.g., paper clips, cubes | Identify the appropriate tool used to measure length (i.e., ruler), weight (i.e., scale), time (i.e., clock, calendar) and temperature (i.e., thermometer) | Select the appropriate units for measuring time, length, weight, and temperature | Select and use the appropriate standard units of measure, abbreviations, and tools for measuring length, weight, and capacity | Use appropriate tools to determine, estimate, and compare units for measurement of weight/mass, area, size of angle (using the benchmark angles 45°, 90°, 180°, 270°, and 360, temperature, length, distance, and volume in metric systems and time in real-life situations. | Select and apply appropriate units for measuring length, mass, volume, and temperature in the metric system | Select appropriate tools and units to determine the measurements needed for calculating perimeter, circumference, area, surface area, and volume | Select and use appropriate tools and units to determine the measurements needed for calculating perimeter, circumference, area, surface area, and volume | NA |

| LEVEL Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------------------|---|--|--|--|---|--|--|---|--|
| Compare, Order, and Estimation | Determine and describe comparisons of length (longer, shorter, the same), mass (heavier, lighter, the same), and capacity (holds more, less, or about the same) using different-shaped or congruent containers, objects or figures. | Compare and order given lengths, capacities, weights, or temperatures that are expressed in the same unit of measure | Compare, order, and describe objects by various measurable attributes for length, weight, and temperature. | Estimate and measure length using fractional parts to the nearest $\frac{1}{2}$ unit in the Metric system. | Estimate and measure a given object to the nearest millimeter | Estimate and measure length to nearest $\frac{1}{2}$ millimeter in the metric system | Estimate and compare corresponding units of measure for temperature, length, and weight/mass in the metric systems | Estimate and compare corresponding units of measure for area and volume/capacity in the metric systems. | Estimate and convert units of measure for mass and capacity within the same measurement system (metric). |

| LEVEL Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------------------|----|----|----|----|---|---|--|---|----|
| Metric Systems | NA | NA | NA | NA | Convert capacity, weight/mass, and length <u>within</u> the metric system of measurement. | Convert units within a given measurement system to include length, weight/mass, and volume. | Convert units within a given measurement system to solve problems. | Convert from one unit to another, perform basic operations, and solve real-world problems using standard (metric) measurements. | NA |

| LEVEL Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------------|----|---|--|---|---|---|---|--|---|
| Formulas | NA | Use a variety of non-standard units to measure length | Use non-standard units to cover a given region | Estimate and measure perimeter and area, using links, tiles, grid paper, geoboards, and dot paper | Describe relationships of rectangular area to numerical multiplication. | Develop, compare, and use formulas to estimate and calculate the perimeter and area of rectangles, triangles, and parallelograms. | Use formulas to determine the radius, diameter, and circumference of a circle and explain the relationship of circumference of a circle to its diameter, linking to pi. | Use formulas and strategies, such as decomposition, to compute the perimeter and area of triangles, parallelograms, trapezoids, the circumference and area of circles, and find the area of more complex shapes. | Use formulas and/or appropriate measuring tools to find length and angle measures (to appropriate levels of precision), perimeter, area, volume, and surface area of polygons, circles, spheres, cones, pyramids, and composite or irregular figures. |

**NCTM Curriculum Focal Points
(Patterns, Functions, and Algebra)
K**

| Level | Focal Points | Connections to Focal Points |
|----------|--------------|---|
| K | | <p>Algebra: Children identify, duplicate, and extend simple number patterns and sequential and growing patterns (e.g., patterns made with shapes) as preparation for creating rules that describe relationships.</p> |

**NCTM Curriculum Focal Points
(Patterns, Functions, and Algebra)
Grade 1**

| Level | Focal Points | Connections to Focal Points |
|---------|--|--|
| Grade 1 | <p>Number and Operations and Algebra: Developing understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts.</p> <p>Children develop strategies for adding and subtracting whole numbers on the basis of their earlier work with small numbers. They use a variety of models, including discrete objects, length-based models (e.g., lengths of connecting cubes), and number lines, to model “part-whole,” “adding to,” “taking away from,” and “comparing” situations to develop an understanding of the meanings of addition and subtraction and strategies to solve such arithmetic problems. Children understand the connections between counting and the operations of addition and subtraction (e.g., adding two is the same as “counting on” two). They use properties of addition (commutativity and associativity) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.</p> | <p>Number and Operations and Algebra:</p> <p>Children use mathematical reasoning, including ideas such as commutativity and associativity and beginning ideas of tens and ones, to solve two-digit addition and subtraction problems with strategies that they understand and can explain. They solve both routine and nonroutine problems.</p> |
| | | <p>Algebra:</p> <p>Through identifying, describing, and applying number patterns and properties in developing strategies for basic facts, children learn about other properties of numbers and operations, such as odd and even (e.g., “Even numbers of objects can be paired, with none left over”), and 0 as the identity element for addition.</p> |

**NCTM Curriculum Focal Points
(Patterns, Functions, and Algebra)
Grade 2**

| Level | Focal Points | Connections to Focal Points |
|---------|--|--|
| Grade 2 | <p>Number and Operations and Algebra: Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction.</p> <p>Children use their understanding of addition to develop quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition (commutativity and associativity). Children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multidigit whole numbers. They select and apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including standard algorithms, for adding and subtracting whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.</p> | <p>Algebra:</p> <p>Children use number patterns to extend their knowledge of properties of numbers and operations. For example, when skip counting, they build foundations for understanding multiples and factors.</p> |

**NCTM Curriculum Focal Points
(Patterns, Functions, and Algebra)
Grade 3**

| Level | Focal Points | Connections to Focal Points |
|----------------|--------------|--|
| Grade 3 | | <p>Algebra:</p> <p>Understanding properties of multiplication and the relationship between multiplication and division is a part of algebra readiness that develops at grade 3. The creation and analysis of patterns and relationships involving multiplication and division should occur at this grade level. Students build a foundation for later understanding of functional relationships by describing relationships in context with such statements as, “The number of legs is 4 times the number of chairs.”</p> |

**NCTM Curriculum Focal Points
(Patterns, Functions, and Algebra)
Grade 4**

| Level | Focal Points | Connections to Focal Points |
|---------|--------------|--|
| Grade 4 | | <p>Algebra:</p> <p>Students continue identifying, describing, and extending numeric patterns involving all operations and nonnumeric growing or repeating patterns. Through these experiences, they develop an understanding of the use of a rule to describe a sequence of numbers or objects.</p> |

**NCTM Curriculum Focal Points
(Patterns, Functions, and Algebra)
Grade 5**

| Level | Focal Points | Connections to Focal Points |
|----------------|---|---|
| Grade 5 | <p>Number and Operations and Algebra: Developing an understanding of and fluency with division of whole numbers.</p> <p>Students apply their understanding of models for division, place value, properties, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multidigit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for dividing whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems. They consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately</p> | <p>Algebra:</p> <p>Students use patterns, models, and relationships as contexts for writing and solving simple equations and inequalities. They create graphs of simple equations. They explore prime and composite numbers and discover concepts related to the addition and subtraction of fractions as they use factors and multiples, including applications of common factors and common multiples. They develop an understanding of the order of operations and use it for all operations.</p> |

Geometry and Measurement and Algebra: Describing three-dimensional shapes and analyzing their properties, including volume and surface area.

Students relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. They understand that a cube that is 1 unit on an edge is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of prisms. As they work with surface area, they find and justify relationships among the formulas for the areas of different polygons. They measure necessary attributes of shapes to use area formulas to solve problems.

**NCTM Curriculum Focal Points
(Patterns, Functions, and Algebra)
Grade 6**

| Level | Focal Points | Connections to Focal Points |
|---------|--|---|
| Grade 6 | <p>Algebra: Writing, interpreting, and using mathematical expressions and equations.</p> <p>Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that make the equation true. They solve simple one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation. They construct and analyze tables (e.g., to show quantities that are in equivalent ratios), and they use equations to describe simple relationships (such as $3x = y$) shown in a table.</p> | <p>Algebra:</p> <p>Students use the commutative, associative, and distributive properties to show that two expressions are equivalent. They also illustrate properties of operations by showing that two expressions are equivalent in a given context (e.g., determining the area in two different ways for a rectangle whose dimensions are $x + 3$ by 5). Sequences, including those that arise in the context of finding possible rules for patterns of figures or stacks of objects, provide opportunities for students to develop formulas.</p> |

**NCTM Curriculum Focal Points
(Patterns, Functions, and Algebra)
Grade 7**

| Level | Focal Points | Connections to Focal Points |
|-------|--|-----------------------------|
| | <p>Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity.</p> <p>Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ($y/x = k$, or $y = kx$) from other relationships, including inverse proportionality ($xy = k$, or $y = k/x$).</p> | |

Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes.

By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders by slicing them, they develop and understand formulas for their volumes ($\text{Volume} = \text{Area of base} \times \text{Height}$). They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students see that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram. They select appropriate two- and three dimensional shapes to model real-world situations and solve a variety of problems (including multistep problems) involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders.

Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations.

Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts (e.g., situations of owing money or measuring elevations above and below sea level), students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. They use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.

**NCTM Curriculum Focal Points
(Patterns, Functions, and Algebra)
Grade 8**

| Level | Focal Points | Connections to Focal Points |
|----------------|---|--|
| Grade 8 | <p>Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations.</p> <p>Students use linear functions, linear equations, and systems of linear equations to represent, analyze, and solve a variety of problems. They recognize a proportion ($y/x = k$, or $y = kx$) as a special case of a linear equation of the form $y = mx + b$, understanding that the constant of proportionality (k) is the slope and the resulting graph is a line through the origin. Students understand that the slope (m) of a line is a constant rate of change, so if the input, or x-coordinate, changes by a specific amount, a, the output, or y-coordinate, changes by the amount ma. Students translate among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations), and they describe how such aspects of a function as slope and y-intercept appear in different representations. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines that intersect, are parallel, or are the same line, in the plane. Students use linear equations, systems of linear equations, linear functions, and their understanding of the slope of a line to analyze situations and solve problems.</p> | <p>Algebra:</p> <p>Students encounter some nonlinear functions (such as the inverse proportions that they studied in grade 7 as well as basic quadratic and exponential functions) whose rates of change contrast with the constant rate of change of linear functions. They view arithmetic sequences, including those arising from patterns or problems, as linear functions whose inputs are counting numbers. They apply ideas about linear functions to solve problems involving rates such as motion at a constant speed.</p> |

| | | |
|--|---|--|
| | <p>Data Analysis and Number and Operations and Algebra: Analyzing and summarizing data sets.</p> <p>Students use descriptive statistics, including mean, median, and range, to summarize and compare data sets, and they organize and display data to pose and answer questions. They compare the information provided by the mean and the median and investigate the different effects that changes in data values have on these measures of center. They understand that a measure of center alone does not thoroughly describe a data set because very different data sets can share the same measure of center. Students select the mean or the median as the appropriate measure of center for a given purpose.</p> | |
|--|---|--|

7.0 PATTERNS, FUNCTIONS, AND ALGEBRA

Algebra is the study of the patterns and symbols that make up the underlying structure of our number system. Algebraic reasoning culminates in the development of a formalized symbolic language that emerges from an early understanding of the “doing and undoing” of number operations. As young children come to understand numbers, patterns, and symbols experientially, they develop the ability to think about numbers and symbols abstractly. Very young children learn number primarily through experiences with counting. As this understanding of counting develops, children begin to use symbols to represent numbers and number sentences to represent operations. In these early years, the development of a symbolic language emerges from experiences with patterns. Students learn about number relationships such as doubles and halves, odds and evens, and multiples of twos, fives, and tens. These experiences with number patterns help students to come to the realization that numbers “behave” in predictable ways. Students represent these insights with number sentences; these number sentences form the beginning stages of symbolic reasoning.

In the intermediate grades, children look at patterns of growth and change. They make graphs and tables to look for trends and to make predictions. Algebraic reasoning is becoming more formal with students developing rules and formulas, such as the cost of a taxi ride or the area of a triangle. Students investigate how variables are related, translating number relationships into rules using variables. In order to apply these rules they solve simple equations using informal methods. Additional experiences with geometry and data reinforce this search for rules and generalizations.

By middle school, students are called upon to move fluently between a variety of models (tables, graphs, and symbolic rules) used to represent variable relationships. They are focused on the study of rates of change and the impact of slope in an equation, a graph, and a table. There is more emphasis on the proper use of symbolic language to describe variable operations. By eighth grade, students are introduced to algebraic transformations and manipulations; they are gaining experiences in demonstrating equivalence in algebraic expressions and generalizing from visual patterns and arithmetic sequences.

These students enter high school with a strong grasp of linear relationships, a formal understanding of the language of linear equations, slope and y-intercept, and a sense of how quadratic and exponential equations differ. High school students explore the increasingly formal, symbolic understandings of families of functions and symbolic transformations. They move fluently among symbolic and graphic representations with a more abstract reasoning system for expressing slope and finite differences and for demonstrating algebraic equivalence. They reason about families of functions and interpret symbolic generalizations about the behavior of standard forms. Symbolic algebra becomes the language of transformation and reasoning used in problem solving and for justification and proof in each of the four strands. Understandings:

Enduring Understandings:

Change is fundamental to understanding functions.

Numbers or objects that repeat in predictable ways can be described or generalized.

An operation can be “undone” by its inverse.

Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.

Essential Questions:

1. How can change be described mathematically?
2. How are patterns of change related to the behavior of functions?
3. How do mathematical models/representations shape our understanding of mathematics?

**7.0 PATTERNS, FUNCTIONS, AND ALGEBRA
STANDARDS and PERFORMANCE INDICATORS**

| | | | | | | | | | |
|------------------------|---|---|---|----------|----------|----------|----------|----------|----------|
| Standard | 7.0 Patterns, Functions, and Algebra Students will use various algebraic methods to analyze, illustrate, extend, and create numerous representations (words, numbers, tables, and graphs) of patterns, functions, and algebraic relations as modeled in practical situations to solve problems, communicate, reason, and make connections within and beyond the field of mathematics. | | | | | | | | |
| Benchmark | 7.1 Patterns, Relations and Functions Students will recognize, describe and develop patterns, relations and functions. Students will be able to: | | | | | | | | |
| Level Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Sorting | Sort objects into groups in one or more ways and identify which attribute was used to sort (size, shape, and color). | Sort and classify objects by one or two attributes in more than one way | Sort, classify, and label objects by three or more attributes in more than one way including color, size, shape, and thickness. | NA | NA | NA | NA | NA | NA |

| Level Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------------|--|--|--|--|--|---|---|---|----|
| Patterns | Identify, reproduce, and extend repeating patterns in visual, auditory, and physical contexts. | <p>Create and explain patterns using concrete objects, numbers, shapes, and colors</p> <p>Formulate, explain, and generalize patterns within and across addition and subtraction</p> | <p>Explain, analyze, and extend repeating and growing patterns</p> <p>Use number patterns to skip count by 2's, 3's, 5's, and 10's</p> | Create, describe, and extend growing and repeating patterns with physical materials and symbols including numbers. | Create, describe, and extend growing and repeating patterns with physical materials and symbols including numbers. | <p>Identify a rule for a pattern involving addition, subtraction, or multiplication</p> <p>Interpret and write a rule for a one operation function table Ex. Adding 3</p> | Create and use tables and charts to extend a pattern in order to describe a rule and find missing terms in a sequence | Use inductive reasoning to extend patterns to predict the nth term (e.g., powers and triangular numbers). | NA |

| Level Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------------------------|--|--|--|--|--|---|---|--|--|
| Identifying Number Patterns | Identify and describe qualitative changes (such as temperature changes – it feels hotter and quantitative changes such as temperature increases five degrees). | Identify, describe, and explain the patterns in repeating situations (adding the same number, e.g., 2, 5, 8, 11, or skip-counting) | Identify patterns of addition and subtraction as represented in charts and tables and in varied forms of skip-counting | Recognize and describe patterns using objects and numbers found in tables, number charts, and charts | Identify and describe patterns resulting from operations involving even and odd numbers (such as even + even = even) | Identify, describe, and represent patterns and relationships in the number system, including triangular numbers and perfect squares | Identify and describe patterns represented by tables, graphs, and sequences | Analyze and describe simple exponential number patterns (e.g., 3, 9, 27 or 3^1 , 3^2 , 3^3) | Distinguish between and explain when real-life numerical patterns are linear/arithmetic (i.e., grows by addition) or exponential/geometric (i.e., grows by multiplication) |

| | | | | | | | | | |
|-------------------------|--|---|--|--|---|--|---|----------|----------|
| Benchmark | 7.2 Algebraic Models Students will represent and analyze mathematical situations and structures using algebraic symbols. Students will be able to: | | | | | | | | |
| Level Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Number Sentences | Use drawings and labels to record solutions of addition and subtraction problems with answers less than or equal to 10 | Select and/or write number sentences to find the unknown in problem-solving contexts involving single-digit addition and subtraction using appropriate labels | Select and/or write number sentences to find the unknown in problem-solving contexts involving two-digit addition and subtraction using appropriate labels | Select and/or write number sentences (equations) to find the unknown in problem-solving contexts involving two-digit times one-digit multiplication using appropriate labels | Select and/or write number sentences (equations) to find the unknown in problem-solving contexts involving two-digit by one-digit division using appropriate labels | Solve problems by finding the next term or missing term in a pattern or function table using real world situations | Determine the rule, output or input; given an input/output model using one operation, write an algebraic expression for the rule and use to identify other input/output values. | NA | NA |

| Level Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------|----|----|----|----|--|---|--|---|---|
| Value of Variables | NA | NA | NA | NA | Determine the rule and explain how change in one variable relates to the change in the second variable, given an input/output model using two operations | Devise a rule for an input/output function table, describing it in words and symbols. | Represent the relationship in an input-output situation using a simple equation, graph, table, or word description | Create tables and graphs to analyze and describe patterns | Find the missing term in a numerical sequence or a pictorial representation of a sequence |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|----|----|--|--|--|---|---|--|--|
| Number Sentences | NA | NA | Model, explain, and identify missing operations and missing numbers in open number sentences including number facts in addition and subtraction. | Model, explain, and solve open number sentences including addition, subtraction, and multiplication facts. | Model, explain, and solve open number sentences including addition, subtraction, multiplication, and division. | Find possible solutions to an inequality involving a variable using whole numbers as a replacement set. | Evaluate formulas and algebraic expressions using whole number values | Evaluate formulas and algebraic expressions for given integer values | Evaluate formulas and algebraic expressions using rational numbers |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------------|----|---|--|--|---|---|---|---|--|
| Variables and Unknowns | NA | Model situations and solve equations that require addition and subtraction of whole numbers; use objects, pictures, and symbols | Model situations and solve equations that involve the addition and subtraction of whole numbers. | Create models for the concept of equality, recognizing that the equal sign (=) denotes equivalent terms such that $4+3=7$, $4+3=6+1$, or $7=5+2$. | Select the solution to an equation from a given set of numbers. | Determine the value of variables in equations and inequalities, justifying the process. | Solve simple equations using guess-and-check, diagrams, properties, or inspection, explaining the process used. | Solve equations that represent algebraic and real-world problems using multiple methods including the real number properties. | Simplify and evaluate numerical and algebraic expressions. |
| | NA | NA | NA | NA | NA | NA | Formulate algebraic expressions, equations, and inequalities to reflect a given situation. | Formulate algebraic expressions, equations, and inequalities to reflect a given situation and vice versa. | Model inequalities (and their solutions) on a number line. |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------|----|----|----|--|--|--|---|---|---|
| Value of Variables | NA | NA | NA | Determine the value of missing quantities or variables within equations or number sentences, and justify the process used. | Determine the value of variables in equations; justify the process used to make the determination. | Solve equations with whole numbers using a variety of methods, including inverse operations, mental math, and guess and check. | Solve and graphically represent equations and inequalities in one variable with integer solutions | Solve and graphically represent equations and inequalities in one variable with integer solutions | Solve and graphically represent equations and inequalities in one variable including absolute value |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--|--|---|--|---|--|---|---|--|----------------------------|
| Number Sentences, Expressions, and Polynomials | Identify and create, compare and describe sets of objects as more, less or equal | Create, compare, and describe sets of objects as greater than, less than, or equal to | Represent mathematical situations using numbers, symbols, and words and complete number sentences with the appropriate words and symbols (+, -, =) | Complete number sentences with the appropriate words and symbols (+, -, >, < =) | Complete number sentences with the appropriate words and symbols (+, -, >, <, ÷, x, =) | Complete number sentences with the appropriate words and symbols including \geq, \leq, \neq | Write simple expressions and equations using variables to represent mathematical situations | Simplify algebraic expressions by combining like terms | Add and subtract binomials |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------------------------|----|----|----|----|----|----|---|--|---|
| Relations and Functions | NA | NA | NA | NA | NA | NA | When given a rule relating two variables create a table and represent the ordered pairs on a coordinate plane | Generate and graph a set of ordered pairs to represent a linear equation | Identify, model, and describe linear functions Translate among verbal descriptions, graphic, tabular, and algebraic representations of mathematical situations |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------------------------------|----|----|----|----|----|----|----|---|--|
| Linear Equations and Inequalities | NA | NA | NA | NA | NA | NA | NA | Model and solve equations using concrete and visual representations | Solve linear equations and inequalities and represent the solution graphically |

| Benchmark | 7.3 Algebraic Representation Students will develop and apply mathematical models to represent and understand quantitative relationships. Students will be able to: | | | | | | | | |
|---|---|---|--|---|---|---|--|--|---|
| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Representing quantitative relationships | NA | Use a chart or table to organize information and to discuss relationships | Use a chart or table to organize information and to understand relationships | Complete a chart or table to organize information and to understand relationships and explain the results | Create a chart or table to organize information and to understand relationships and explain the results | Draw conclusions and make predictions from models, tables and line graphs | Create algebraic expressions that correspond to real-world situations; use the expressions to solve problems | Solve problems by creating an input/output function table(including, but not limited to, spreadsheets) to predict future values, given a real-world situation involving rational numbers. | Identify proportional relationships in real-world situations, then find and select an appropriate method to determine the solution; justify the reasonableness of the solution. |

| Benchmark | 7.4 Analysis of Change Students will analyze change in various contexts. Students will be able to: | | | | | | | | |
|-----------------|--|--|--|---|---|--|---|--|---|
| Level Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Change | Describe qualitative change Ex. Changes in seasons, temperature, height, etc | Describe quantitative changes Ex. Compare a wide variety of measurements over time (e.g., students' heights, plant growth). | Compare and contrast the attribute changes over time in two or more qualities. | Identify real situations and events that show change Ex. Which day had the greatest change in temperature? | Identify, describe and generalize relationships in which quantities change proportionally | Use a variety of methods to compare and describe situations involving constant and/or varying rates of change. | Describe how changes in one quantity or variable result in changes in another | Use unit rates (e.g., miles per hour, words per minutes) to solve problems | Compare linear relationships to non-linear relationships; |

**NCTM Focal Points
(Geometry)
K**

| Level | Focal Points | Connection to the Focal Points |
|---|---|--|
| <p style="text-align: center;">K</p> | <p>Geometry: Describing shapes and space</p> <p>Children interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of shapes, such as squares, triangles, circles, rectangles, (regular) hexagons, and (isosceles) trapezoids presented in a variety of ways (e.g., with different sizes or orientations), as well as such three-dimensional shapes as spheres, cubes, and cylinders. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.</p> | <p>Geometry:</p> <p>Children integrate their understandings of geometry, measurement, and number. For example, they understand, discuss, and create simple navigational directions (e.g., “Walk forward 10 steps, turn right, and walk forward 5 steps”).</p> |

**NCTM Focal Points
(Geometry)
Grade 1**

| Level | Focal Points | Connection to the Focal Points |
|----------------|---|--------------------------------|
| Grade 1 | <p>Geometry: Composing and decomposing geometric shapes.</p> <p>Children compose and decompose plane and solid figures (e.g., by putting two congruent isosceles triangles together to make a rhombus), thus building an understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine figures, they recognize them from different perspectives and orientations, describe their geometric attributes and properties, and determine how they are alike and different, in the process developing a background for measurement and initial understandings of such properties as congruence and symmetry.</p> | |

**NCTM Focal Points
(Geometry)
Grade 2**

| Level | Focal Points | Connection to the Focal Points |
|----------------|--------------|---|
| Grade 2 | | <p>Geometry and Measurement:</p> <p>Children estimate, measure, and compute lengths as they solve problems involving data, space, and movement through space. By composing and decomposing two-dimensional shapes (intentionally substituting arrangements of smaller shapes for larger shapes or substituting larger shapes for many smaller shapes), they use geometric knowledge and spatial reasoning to develop foundations for understanding area, fractions, and proportions.</p> |

**NCTM Focal Points
(Geometry)
Grade 3**

| Level | Focal Points | Connection to the Focal Points |
|----------------|--|--------------------------------|
| Grade 3 | <p>Geometry: Describing and analyzing properties of two-dimensional shapes.</p> <p>Students describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes. Students investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space and the use of those attributes and properties in solving problems, including applications involving congruence and symmetry.</p> | |

| NCTM Focal Points (Geometry) Grade 4 | | |
|---|---------------------|--|
| Level | Focal Points | Connection to the Focal Points |
| Grade 4 | | Geometry: Students extend their understanding of properties of two-dimensional shapes as they find the areas of polygons. They build on their earlier work with symmetry and congruence in grade 3 to encompass transformations, including those that produce line and rotational symmetry. By using transformations to design and analyze simple tilings and tessellations, students deepen their understanding of two-dimensional space. |

**NCTM Focal Points
(Geometry)
Grade 5**

| Level | Focal Points | Connection to the Focal Points |
|----------------|--|--------------------------------|
| Grade 5 | <p>Geometry and Measurement and Algebra: Describing three-dimensional shapes and analyzing their properties, including volume and surface area.</p> <p>Students relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. They understand that a cube that is 1 unit on an edge is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of prisms. As they work with surface area, they find and justify relationships among the formulas for the areas of different polygons. They measure necessary attributes of shapes to use area formulas to solve problems.</p> | |

**NCTM Focal Points
(Geometry)
Grade 6**

| Level | Focal Points | Connection to the Focal Points |
|---------|--------------|--|
| Grade 6 | | <p>Measurement and Geometry:</p> <p>Problems that involve areas and volumes, calling on students to find areas or volumes from lengths or to find lengths from volumes or areas and lengths, are especially appropriate. These problems extend the students' work in grade 5 on area and volume and provide a context for applying new work with equations.</p> |

**NCTM Focal Points
(Geometry)
Grade 7**

| Level | Focal Points | Connection to the Focal Points |
|---------|--|--|
| Grade 7 | <p>Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity.</p> <p>Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ($y/x = k$, or $y = kx$) from other relationships, including inverse proportionality ($xy = k$, or $y = k/x$).</p> | <p>Measurement and Geometry:</p> <p>Students connect their work on proportionality with their work on area and volume by investigating similar objects. They understand that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related. Students apply their work on proportionality to measurement in different contexts, including converting among different units of measurement to solve problems involving rates such as motion at a constant speed. They also apply proportionality when they work with the circumference, radius, and diameter of a circle; when they find the area of a sector of a circle; and when they make scale drawings.</p> |

Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes.

By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders by slicing them, they develop and understand formulas for their volumes ($\text{Volume} = \text{Area of base} \times \text{Height}$). They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students see that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram. They select appropriate two- and three dimensional shapes to model real-world situations and solve a variety of problems (including multistep problems) involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders.

**NCTM Focal Points
(Geometry)
Grade 8**

| Level | Focal Points | Connection to the Focal Points |
|---------|--|---|
| Grade 8 | <p>Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angle.</p> <p>Students use fundamental facts about distance and angles to describe and analyze figures and situations in two- and three-dimensional space and to solve problems, including those with multiple steps. They prove that particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lines. Students apply this reasoning about similar triangles to solve a variety of problems, including those that ask them to find heights and distances. They use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degrees, and they apply this fact about triangles to find unknown measures of angles. Students explain why the Pythagorean theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane to measure lengths and analyze polygons and polyhedra.</p> | <p>Geometry:</p> <p>Given a line in a coordinate plane, students understand that all “slope triangles” triangles created by a vertical “rise” line segment (showing the change in y), a horizontal “run” line segment (showing the change in x), and a segment of the line itself—are similar. They also understand the relationship of these similar triangles to the constant slope of a line.</p> |

8.0 GEOMETRY

Geometry (derived from “geo” and “metric” so, literally, the “measure of the earth”) is comprised of several important but distinct strands, all of which should be developed across grades K–12. In very young children, we build upon intuitive understandings that, in combination, make up the sense of space. These begin with simple ideas of relative position (in front of, “over there”) and culminate with two and then three dimensional coordinate geometry. Gradually, distance measurement becomes an important component of this spatial sense.

Early on, movement in both two and three dimensions becomes another component of a child’s developing spatial sense. This movement is first formalized as simple “transformations” of position accessible in the primary grades—flips, slides, and turns—and then given more formal definition and deeper exploration in the middle grades as reflections, translations, and rotations. Ultimately, transformations of position, size, and shape are embedded within the coordinate plane and described algebraically using matrices.

Visual representations play a pivotal role in supporting reasoning about geometric objects and properties. Because most representational systems, from a “flat” piece of paper to a computer screen, are two dimensional, conventions for representing three dimensional objects in two dimensions, “projections,” must be developed. A range of representational systems from paper folding (useful for emphasizing instances of congruence, for example) to Geo Logo (students must input side length and angle measure) to computer based geometric “supposers,” which support the making and testing of conjectures about geometric figures, find application across the grades.

Natural three dimensional objects can be abstracted into perfect forms, tree trunks as the prototypes of the cylinder, the arc of an ancient slingshot whirling overhead as the ideal of the circle with a given center and radius. These idealized geometrical forms are then categorized, their elements defined, and their relationship to one another determined. The classification of two and three dimensional geometric objects proceeds from an essentially perceptual and inductive basis—rectangles “look the same”—to an understanding of the set of characteristics that these objects share, for example, all rectangles have four right angles and opposite sides congruent and parallel. A primarily inductive reasoning about geometric characteristics transitions into a more deductive understanding of how one class of objects relates to another, for example, a square is a rectangle with an added constraint.

The development of a fully axiomatic system for the classification of geometric objects provides an important opportunity for high school students to develop deductive reasoning and proof.

ENDURING UNDERSTANDINGS:

1. Two- and three-dimensional objects can be described, classified, and analyzed by their attributes.
2. An object in a plane or in space can be oriented in an infinite number of ways while maintaining its size or shape.
3. An object's location on a plane or in space can be described quantitatively.
4. Linear measure, area, and volume are fundamentally different but may be related to one another in ways that permit calculation of one given the other

ESSENTIAL QUESTIONS:

1. Why do we compare contrast and classify objects?
2. How do decomposing and recomposing shapes help us build our understand of mathematics?
3. How can transformations be described mathematically?

8.0 Geometry
STANDARDS AND PERFORMANCE INDICATORS

| | | | | | | | | | |
|--------------------------------|---|---|--|---|---|---|--|--|--|
| Standard | 8.0 GEOMETRY The student will develop an understanding of geometric concepts and relationships as the basis for geometric modeling and reasoning to solve problems involving one-, two-, and three-dimensional figures. | | | | | | | | |
| Benchmark | 8.1 Geometric Properties Students will analyze characteristics and properties of 2 and 3 dimensional geometric shapes and develop mathematical arguments about geometric relationships. Students will be able to: | | | | | | | | |
| LEVEL Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Two-Dimensional Figures | Identify two-dimensional shapes, i.e., circle, triangle, rectangles, and squares, regardless of size or orientation | Name, sort, and sketch two-dimensional shapes (circles, triangles, rectangles including squares) regardless of orientation. | Describe, sketch, and compare two-dimensional shapes (rhombus, square, triangle, trapezoid, rectangle, pentagon, hexagon, octagon, and decagon) regardless of orientation. | Describe, compare, analyze, and classify two-dimensional shapes by sides and angles | Analyze and describe the similarities and differences between and among two dimensional geometric shapes, figures, and models using mathematical language | Analyze and describe the characteristics of symmetry relative to classes of polygons (parallelograms, triangles, etc.). | Identify, classify, compare, and draw regular and irregular quadrilaterals | Identify, classify, and compare, and draw regular and irregular polygons | Use two-dimensional representations (nets) of three-dimensional objects to describe objects from various perspectives. |

| LEVEL Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------------------|--|--|---|--|---|---|--|---|---|
| Three-Dimensional Figures | Identify three-dimensional figures in the environment. | Identify, name, and describe three-dimensional objects i.e. cubes and spheres, regardless of size or orientation | Identify, classify, and sort basic geometric figures by shape, size, and geometric attributes. E.g. cube, sphere, and cylinder, prism, pyramid, and cone. | Identify, describe, and classify: cube, sphere, prism, pyramid, cone, and cylinder in terms of the number and shape of faces, edges, and vertices. | Analyze the relationship between three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders and their two-dimensional nets. | Predict and describe the effects of combining, dividing, and changing shapes into other shapes. | Construct three-dimensional figures using manipulatives and generalize the relationships among vertices, faces, and edges (such as Euler's Formula). | Construct two-dimensional representations of three-dimensional objects. | Classify and compare three-dimensional shapes using their properties. |

| Benchmark | 8.2 Transformation of Shapes Students will apply transformations and the use of symmetry to analyze mathematical situations | | | | | | | | |
|---------------------------|---|---|--|--|---|--|---|--|--|
| Level Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Congruency and Similarity | Use relative position words including before/after, far/near, and over/under to place objects | Use position words down/up, left/right, top/bottom, and between/middle to describe the relative location of objects | Identify congruent and similar shapes (circles, triangles, and rectangles including squares) | Identify and create shapes that have lines of symmetry | Identify shapes that are congruent, similar, and/or symmetric using a variety of methods including transformational motions | Represent concepts of congruency, similarity and/or symmetry using a variety of methods including dilation and transformational motion | Generalize the relationship between line symmetry and rotational symmetry for two-dimensional shapes. | Analyze geometric properties and the relationships among the properties of triangles, congruence, similarity, and transformations to make deductive arguments. | Apply the properties of equality and proportionality To find missing attributes of congruent or similar shapes |

| LEVEL Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------|--|---|--|---|--|---|---|--|--|
| Transformations | Describe the location of one object relative to another object using words such as <i>in, out, over, under, above, below, between, next to, behind, and in front of.</i> | Use the directional words left, and right to describe movement. | Use the positional and directional terms north, south, east, and west to describe location and movement. | Describe the transformational motions of geometric figures (translation /slide, reflection/flip, and rotation/turn) | Compare figures to determine congruence using geometric transformations (motions), such as reflections (flips), rotations (turns), and translations (slides).. | Predict the results of multiple transformations on a geometric shape when combinations of translation, reflection, and rotation are used. | Compare, classify, and construct transformations (reflections, translation, and rotations). | Perform transformations (rigid and non-rigid motions) on two-dimensional figures using the coordinate plane. | Draw the results of a combination of transformations in the coordinate plane; i.e., reflections, rotations, and translations |

| LEVEL Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|----------------------|--|---|--|---|---|---|--|--|---|
| Representing Figures | Represent two-dimensional geometric shapes | Compose and decompose common two-dimensional figures. | Predict the results of combining and subdividing polygons and circles. | Analyze the results of combining and subdividing circles, triangles, quadrilaterals, pentagons, hexagons, and octagons. | Represent the two-dimensional shapes trapezoids, rhombuses, and parallelograms and the three-dimensional shapes cubes, rectangular prisms, and cylinders. | Predict and describe the effects of combining, dividing, and changing shapes into other shapes. | Make a two-dimensional drawing of a three-dimensional figure | Make a model of a three-dimensional figure from a two-dimensional drawing. | Use two-dimensional representations of three-dimensional objects to visualize and solve problems; e.g., those involving surface area and volume |

| LEVEL Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------|----|----|----|----|---|---|---|---|---|
| Triangles | NA | NA | NA | NA | Identify, classify and draw triangles based on their properties | Describe the characteristics of a right triangle | Describe the characteristics of right, acute, obtuse, scalene, equilateral, and isosceles triangles | Classify triangles based on side and angle measurements; | Determine the measure of the missing side of a right triangle. |
| | NA | NA | NA | NA | NA | Determine the degrees of the interior angles of triangles | Determine the measure of missing angles of triangles based on the Triangle Sum Theorem | Determine the measure of the missing side of a right triangle | Find and verify the sum of the measures of interior angles of triangles |

| LEVEL Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------|----|----|----|----|----|----|----|--|---|
| | NA | NA | NA | NA | NA | NA | NA | Create an argument using the Pythagorean Theorem principles to show that a triangle is a right triangle. | Explain the Pythagorean Theorem and apply it to solve routine and non-routine problems. |

| Level Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------------------------------------|----|----|----|--|---|---|--|--|--|
| Lines, Angles, and their properties | NA | NA | NA | Identify, draw, and describe horizontal, vertical, and oblique lines | Identify, draw, label, and describe points, line segments, rays, and angles | Identify, draw, label, and describe planes, parallel lines, intersecting lines, and perpendicular lines | Model slope (pitch, angle of inclination) using concrete objects and practical examples | Determine slope of a line, midpoint of a segment and the horizontal and vertical distance between two points using coordinate geometry | Calculate slope, midpoint, and distance using equations and formulas |

| LEVEL Indicator | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|--------------------|----|----|----|----|--|---|--|--|---|
| Angles | NA | NA | NA | NA | Identify, draw, and classify angles, including straight, right, obtuse, and acute. | Identify the attributes of an angle and draw angles using protractors | Identify relationships between pairs of angles; i.e., adjacent, vertical, complementary, and supplementary | Find missing angle measurements for parallel lines cut by a transversal (s) and for a vertex of a polygon. | Locate and identify angles formed by parallel lines cut by a transversal (s) (e.g., adjacent, vertical, complementary, supplementary, corresponding, alternate interior, and alternate exterior). |

| | | | | | | | | | | |
|---------------------|------------------------------------|--|---|--|---|---|---|--|--|---|
| Benchmark | | 8.3 Coordinate Geometry Students will specify locations and describe spatial relationships using coordinate geometry and other representational systems. | | | | | | | | |
| LEVEL Indicators | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Coordinate Geometry | Descriptions Spatial Relationships | Use the positional words near, far, below, above, beside, next to, across from, and between to describe the location of an object | Use the positional and directional terms north, south, east, and west to describe location and movement | Use ordered pairs to identify the locations of points in a grid; e.g., A-10 on a map | Use coordinate s to give or follow directions from one point to another on a map or grid. | Locate and label ordered pairs in the first quadrant of the coordinate plane. | Graph coordinate s representin g geometric shapes in the first quadrant | Draw and label the component s of the coordinate plane; i.e., coordinate s, quadrants, origin, x- and y-axes | Represent shapes using coordinate geometry | Apply strategies and procedures to find the coordinate s of the missing vertex of a square, rectangle, or right triangle when given the coordinate s of the polygon's other vertices. |

| Benchmark | 8.4 Visualization and Geometric Models Students will use visualization, spatial reasoning and geometric modeling. | | | | | | | | |
|------------------|--|----|----|----|----|----|---|---|--|
| LEVEL Indicators | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | NA | NA | NA | NA | NA | NA | Construct circles, angles, and triangles based on given measurements using a variety of methods and tools including compass, straight edge, paper folding, and technology | Build and sketch three-dimensional solids; e.g., using nets, manipulatives. Construct and identify congruent angles, parallel lines, and perpendicular lines | Construct geometric figures using a variety of tools |

| | | | | | | | | | |
|---|----------------------------------|--|--|--|--|---|--|--|--|
| Mathematical reasoning Logic | Put events in a logical sequence | Identify what comes next in a step by step story or event sequence | Sort and classify objects by two or more attributes. | Use the quantifiers all, some, and none to describe the characteristics of a set | Use the connectors and, or, and not to describe the members of a set | Represent relationships using Venn diagrams | Identify counterexamples to disprove a conditional statement | Make and test conjectures to explain observed mathematical relationships and to develop logical arguments to justify conclusions | Represent logical relationships using conditional statements |
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**NCTM Focal Points
(Data Analysis and Probability)
K**

| Level | Focal Points | Connections to Focal Points |
|----------|--------------|---|
| K | | <p>Data Analysis:</p> <p>Children sort objects and use one or more attributes to solve problems. For example, they might sort solids that roll easily from those that do not. Or they might collect data and use counting to answer such questions as, “What is our favorite snack?” They re-sort objects by using new attributes (e.g., after sorting solids according to which ones roll, they might re-sort the solids according to which ones stack easily).</p> |

**NCTM Focal Points
(Data Analysis and Probability)
Grade 1**

| Level | Focal Points | Connections to Focal Points |
|----------------|--------------|--|
| Grade 1 | | <p>Measurement and Data Analysis:</p> <p>Children strengthen their sense of number by solving problems involving measurements and data. Measuring by laying multiple copies of a unit end to end and then counting the units by using groups of tens and ones supports children's understanding of number lines and number relationships. Representing measurements and discrete data in picture and bar graphs involves counting and comparisons that provide another meaningful connection to number relationships.</p> |

**NCTM Focal Points
(Data Analysis and Probability)
Grade2**

| Level | Focal Points | Connections to Focal Points |
|---------|--------------|-----------------------------|
| Grade 2 | | |

**NCTM Focal Points
(Data Analysis and Probability)
Grade 3**

| Level | Focal Points | Connections to Focal Points |
|---------|--------------|---|
| Grade 3 | | Data Analysis: Addition, subtraction, multiplication, and division of whole numbers come into play as students construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems. |

**NCTM Focal Points
(Data Analysis and Probability)
Grade 4**

| Level | Focal Points | Connections to Focal Points |
|---------|--------------|--|
| Grade 4 | | Data Analysis: Students continue to use tools from grade 3, solving problems by making frequency tables, bar graphs, picture graphs, and line plots. They apply their understanding of place value to develop and use stem-and-leaf plots. |

**NCTM Focal Points
(Data Analysis and Probability)
Grade 5**

| Level | Focal Points | Connections to Focal Points |
|---------|--------------|---|
| Grade 5 | | Data Analysis: Students apply their understanding of whole numbers, fractions, and decimals as they construct and analyze double-bar and line graphs and use ordered pairs on coordinate grid |

**NCTM Focal Points
(Data Analysis and Probability)
Grade 6**

| Level | Focal Points | Connections to Focal Points |
|---------|--------------|-----------------------------|
| Grade 6 | | |

**NCTM Focal Points
(Data Analysis and Probability)
Grade 7**

| Level | Focal Points | Connections to Focal Points |
|---------|--------------|---|
| Grade 7 | | Data Analysis: Students use proportions to make estimates relating to a population on the basis of a sample. They apply percentages to make and interpret histograms and circle graphs. |

NCTM Focal Points
(Data Analysis and Probability)
Grade 8

| Level | Focal Points | Connections to Focal Points |
|----------------|---|---|
| Grade 8 | <p>Data Analysis and Number and Operations and Algebra: Analyzing and summarizing data sets.</p> <p>Students use descriptive statistics, including mean, median, and range, to summarize and compare data sets, and they organize and display data to pose and answer questions. They compare the information provided by the mean and the median and investigate the different effects that changes in data values have on these measures of center. They understand that a measure of center alone does not thoroughly describe a data set because very different data sets can share the same measure of center. Students select the mean or the median as the appropriate measure of center for a given purpose.</p> | <p>Data Analysis:</p> <p>Building on their work in previous grades to organize and display data to pose and answer questions, students now see numerical data as an aggregate, which they can often summarize with one or several numbers. In addition to the median, students determine the 25th and 75th percentiles (1st and 3rd quartiles) to obtain information about the spread of data. They may use box-and-whisker plots to convey this information. Students make scatterplots to display bivariate data, and they informally estimate lines of best fit to make and test conjectures.</p> |

9.0 DATA ANALYSIS and PROBABILITY

In the primary grades, students collect data about important questions that involve choices, “What do you like better, whales or eagles, pizza or ice cream?” They organize the results in pictographs with visuals that physically represent the choices (a picture of a whale or an ice cream cone is placed on the graph and represents the choice). Students construct their own data displays and use them to answer questions such as “which is less and which is more.” As the study of data progresses, students conduct experiments and surveys that include more categories and choices. Students are called upon to order their data using tables and tallies, and ultimately to represent both categorical and numerical data. Their use of graphs expands to include dot/line plots, and they begin to think more globally about the shape of the distribution of the data. Student experiences with probability are also based on common experiences and shared understandings of the world around them such as “how likely is it that the sun will shine tomorrow or is it likely that we will have rain tomorrow?” Their thinking about probability is categorical (likely vs. unlikely or possible vs. impossible) rather than numerical.

As they enter the intermediate grades, students begin to think more critically about one variable data sets. They grapple with the need to determine a typical, or average, value from a set of data, which leads to understanding different measures of central tendency. They pose questions and design experiments that require the collection of numerical data. In addition to pictographs, bar graphs, and line plots, they begin to use circle graphs to represent and analyze data. Student experiences with probability expand and become more formalized as they conduct experiments to determine whether a game with spinners or number cubes is “fair.” They develop ways to make organized lists and sample spaces to represent all possible outcomes, primarily focused on fair games, and use statistical tools such as tallies and relative frequencies to solve probability problems.

In middle school, students are able to take two variable data sets and compare those using measures of central tendency, stem and leaf plots, histograms and box and whisker plots. They can demonstrate that there are uses and misuses of data and how measures of central tendency may be misleading. Students are more aware of the impact of scaling on graphs and can demonstrate their understanding by building effective displays of data. They use scatter plots to investigate the relationship between two variables and use lines of best fit, where appropriate, to make predictions. Middle school experiences with probability become more sophisticated and involve the use of simulations. Students think about how to select a “fair” (random) sample. They develop an understanding of theoretical probability and how it relates to experimental probability. Well organized tree diagrams and lists are used to justify calculations of theoretical probabilities. Conceptual and procedural knowledge of probability now requires fluency with rational number and a strong grounding in proportional reasoning.

In high school, while conceptual foundations for both statistics and probability continue to be developed using data drawn from experiments, students are expected to generalize and formalize their knowledge. Scatter plots and mathematical models enable predictions to be made, while mathematical reasoning addresses possible limitations for the predictions. Experimental probabilities support conjectures regarding theoretical probabilities, which can now be computed using formulas. Statistics and probability are woven together through investigations of distributions of data, including normal distributions.

ENDURING UNDERSTANDINGS:

1. The question to be answered determines the data to be collected and how best to collect it.
2. Basic statistical techniques can be used to analyze data in the workplace.
3. The probability of an event can be used to predict the probability of future events.

ESSENTIAL QUESTIONS:

1. What is average?
2. What makes a data representation useful?
3. How does my sample affect confidence in my predication?
4. What is fair?

| LEVEL | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------|---|---|--|--|---|---|---|--|---|
| Summary | At this level students, sort objects and represent and interpret data using picture graphs. | At this level, students begin collecting data, using tally marks to represent the data. They read and interpret tally charts and picture graphs as sources of information | At this level, students collect data, and use tables, pictographs, and bar graphs to represent the data. | At this level, students collect data and use graphical representations including number lines, frequency tables, and pictographs to represent data | At this level, students represent data on a number line and in frequency tables, interpret data graphs to answer questions, and summarize the results of probability experiments in an organized way. | At this level, students use the mean, median, mode, and range to describe data sets. They further develop the concept of probability, recording probabilities as fractions between 0 and 1 and linking these to levels of certainty about the events described. | At this level, students learn how to display data in frequency tables and in stem-and-leaf plots. They compare the mean, median, and mode. They find probabilities for compound events and write them as fractions, decimals, and percentages. They also estimate the probabilities of future events. | At this level, students learn how to display data in bar, line, and circle graphs and in stem-and-leaf plots. They analyze data displays to find whether they are misleading and analyze the wording of survey questions to tell whether these could influence the results. They find the probability of disjoint events. They also find the number of arrangements of objects using a tree diagram. | At this level, students evaluate whether claims based on data are reasonable and employ various sampling methods, analyzing their strengths and weaknesses. They understand the concepts of the median and quartiles and use these measures to draw and analyze box-and-whisker plots. They represent and analyze two-variable data using scatterplots. They understand the concept of equally likely events and use it to find probabilities. They also find the number of arrangements of objects using the Basic Counting Principle. |

**9.0 DATA ANALYSIS and PROBABILITY
STANDARDS AND PERFORMANCE INDICATORS**

| | | | | | | | | | | |
|--------------------------------------|---|--|--|---|---|--|---|---|--|--|
| Standard | | 9.0 DATA ANALYSIS and PROBABILITY Students will develop an understanding of Data Analysis and Probability by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions and to present convincing arguments; and to model mathematical situations to determine the probability. | | | | | | | | |
| Level | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Understandings by grade level | | Collect, organize, and interpret data. | Collect, organize, and interpret data in graphical form. | Organize and interpret data in graphical form. | Interpret and analyze data. Explore basic concepts of probability. | Formulate and analyze data. Evaluate inferences and predictions. | Interpret and analyze data and make predictions. | Organize, interpret, analyze, and display data to predict trends. | Organize and interpret data. Analyze data to make predictions. | Interpret, organize, and make predictions about a variety of data using concepts of probability. |
| Benchmark | | 9.1 Data Representation Students will formulate questions that can be addressed with data and collect, organize and display relevant data to answer them. Students will be able to: | | | | | | | | |
| Level | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | Organizing and Representing Data | Collect and organize data by counting and using tally marks and other symbols. | Gather data and represent data using tallies, tables, picture graphs, and bar-type graphs. | Use tables, pictographs, and bar graphs to represent data | Use graphical representations including number lines, frequency tables, and pictographs to represent data | Use a variety of graphical representations including frequency tables and plots to organize and represent data | Use a variety of graphical representations including line graphs, stem-and-leaf plots, histograms, and box-and-whisker plots to organize and represent data | Use a variety of graphical representations including circle graphs and scatter plots to organize and represent data | Use the appropriate graphical representations to organize and represent data | Use appropriate graphical representations to organize, display, and read data . |

| Level | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------|-----------------------|--|--|--|---|---|--|--|--|--|
| | Formulating Questions | Identify a real life situation to gather data over time; | Identify a real life situation to gather data over time; | Identify real life situations to gather data over time; | Pose questions that can be used to guide data collection, organization, and representation | Pose questions that can be used to guide data collection, organization, and representation | Pose questions that can be used to guide the collection of categorical and numerical data | Formulate questions that guide the collection of data | Formulate questions that guide the collection of data | Formulate questions and design a study that guides the collection of data |
| Benchmark | | 9.2 Data Analysis Students will select and use appropriate statistical methods to analyze data. Students will be able to: | | | | | | | | |
| Level | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | Interpreting Data | Describe data by using mathematical language such as more than, less than, etc | Analyze and interpret data by using mathematical language such as more than, less than, etc. | Interpret data presented in circle, line, and bar graphs and answer questions about the displayed situation. | Compare data and interpret quantities represented on tables and different types of graphs (line plots, pictographs, and bar graphs), make predictions, and solve problems based on the information. | Compare data and interpret quantities represented on tables and graphs including line graphs, bar graphs, frequency tables, and stem-and-leaf plots to make predictions and solve | Compare data and interpret quantities represented on tables and graphs, including line graphs, stem-and-leaf plots, histograms, and box-and-whisker plots to make predictions, and solve problems based on the information | Interpret and explain line graphs, double bar graphs, frequency plots, stem-and-leaf plots, histograms, and box-and-whisker plots. | Interpret graphical representations of data to describe patterns, trends, and data distributions | Draw inferences, make conjectures and construct convincing arguments involving different effects that changes in data values have on measures of central tendency misuses of statistical or numeric information, based on data analysis of same and different sets of data |

| Level | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|----------------------------|---|----|----|--|---|---|---|---|---|
| | Central Tendency | NA | NA | NA | NA | Determine or calculate the mode, mean/average, and range for a data set | Collect and analyze data using mean, median and mode to determine the best statistical measure. | Analyze and solve application problems involving measures of central tendency (mean, median, mode) and dispersion (range) from data, graphs, tables, and experiments use appropriate technology to compare two sets of data | Determine which measure of central tendency (mean, median) provides the most useful information in a given context. | Select and apply appropriate measures of data distribution using interquartile range and central tendency |
| Benchmark | | 9.3 Inferences and Predictions Students will develop and evaluate inferences and predictions that are based on data. Students will be able to: | | | | | | | | |
| | Inferences and Predictions | NA | NA | NA | Record results of activities involving chance (e.g., coin flips, dice rolls) and make reasonable predictions based upon data | Make predictions and draw conclusions from simple experiments | Make predictions and draw conclusions based on data collected from a sample group | Analyze various representations of a set of data to draw conclusions and make predictions. | Interpolate and extrapolate from data to make predictions for a given set of data | Formulate reasonable inferences and predictions through interpolation and extrapolation of data to solve practical problems |

| Level | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------------------------|----|----|----|----|----|----|----|---|--|
| Experiments and samples | NA | NA | NA | NA | NA | NA | NA | Identify a real life situation using statistical measures (mean, median, mode, range, outliers) overtime, make a hypothesis as to the outcome; Design and implement a method to collect, organize and analyze data to make a conclusion | Analyze problem situations, games of chance, and consumer applications using random and non-random samplings to determine probability, make predictions, and identify sources of bias. |

| Benchmark | | 9.4 Probability Students will understand and apply basic concepts of probability. Students will be able to: | | | | | | | | |
|-------------|-------------------------------|---|---|---|---|---|---|--|---|--|
| Level | | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Probability | Experimental Probability | Use chance devices like spinners and dice to explore concepts of probability and use tallies to record results in a table, make predictions (More likely, less likely, equally likely) based on results. | Conduct simple experiments, record data on a tally chart or table and use the data to predict which of the events is more likely or less likely to occur if the experiment is repeated. | Conduct simple experiments with more than two outcomes and use the data to predict which event is more, less, or equally likely to occur if the experiment is repeated. | Develop and conduct grade-appropriate experiments using concrete objects (e.g. counters, number cubes, spinners) to determine the likeliness of events and list all outcomes. | Design and conduct a simple probability experiment using concrete objects, examine and list all possible combinations using a tree diagram, represent the outcomes as a ratio and present the results | Construct a sample space and make a hypothesis as to the probability of a real life situation overtime, test the prediction with experimentation, and present conclusions | Perform simple probability events using manipulatives; predict the outcome given events using experimental and theoretical probability; express experimental and theoretical probability as a ratio, decimal or percent. | Determine theoretical probability of an event, make and test predictions through experimentation. | Compare the experimental and theoretical probability of a given situation (including compound probability of a dependent and independent event). |
| Probability | Permutations and Combinations | NA | NA | NA | NA | NA | Determine combinations and permutations of given real-world situations by multiple strategies, including creating lists. | Determine combinations and permutations by constructing sample spaces (e.g., listing, tree diagrams, frequency distribution tables). | Determine and explain whether a real-world situation involves permutations or combinations, then use appropriate technology to solve the problem. | Find the number of combinations possible in mathematical and practical situations |

Glossary for Mathematics

Absolute Value.. A number's distance from zero on a number line. The absolute value of 2 is equal to the absolute value of -2.

Acute Angle.. An angle that measures less than 90 degrees.

Algebraic Expression... a group of numbers, symbols, and variables that express a single or series of operations; mathematical phrase with one or more terms, one or more variables

Algorithm.. A rule or procedure for completing or solving a certain type of problem.

Amortization Table.. A table used to display monthly payment costs (principal + interest) based on loan interest rate and the amount of time that will be used to repay the loan.

Angle of Inclination.. The positive angle, less than 180 degrees, that measures the steepness of the slope or the rate at which the height of the line changes.

Array.. a rectangular arrangement of objects in rows and columns (no gaps and no overlaps)

Arc.. Any section of the circumference of a circle.

Area.. The size of a region measured in number of square units (i.e., inches squared, square feet)

Associative ..Refers to the fact that when adding or multiplying three or more numbers, any order can be used and the answer will be the same.

Attribute... A characteristic of an object, such as color, shape, size, etc.

Binomials ..The algebraic expressions with exactly two terms ($2x + 3y$).

Box and Whisper Plot... A graphical method used to display the middle (median) of a set of data, the middle (median) of each half of that data, and the extremes of the data.

Calculate (Compute).. The process of adding, subtracting, multiplying, dividing or finding the square root of an equation/problem.

Capacity.. The maximum amount of liquid a container can hold.

Chord...A straight line segment that connects two sides of a circle, but does not go through the exact center.

Commutative... The order in which two numbers are added or multiplied does not change the sum.

Complementary... Two angles whose measures sum exactly 90 degrees.

Composite Number... A whole number that is the result of at least two numbers (with "1" not one of the numbers) being multiplied together and that can be broken down into factors (i.e., $20 = 2 * 10$).

Concrete Objects... physical objects used to represent mathematical situations

Congruent.. Figures that have the same size and shape.

Conjecture.. An idea or theory that has not been proved.

Converse.. A particular type of logic or "truth statement" that describes when the two terms of the statement can be interchanged and the statement remain true (If I am alive, then I will die. If I will die, then I am alive).

Convert ..To change from one form to another (i.e., $1/2 = 50\%$).

Coordinate Grid.. A flat surface divided into square units that are numbered and can be used to locate items on the grid.

Cylinder.. A three-dimensional figure shaped like a soup can.

Decimal Number System...A place value number system based on groupings by powers of ten

Deductive Reasoning... The process of reasoning that starts from statements accepted as true and applied to a new situation to reach a conclusion (i.e., if $5 + 4 = 9$ and $6 + 3 = 9$ then $5 + 4 = 6 + 3$).

Denominator.. The number that is the bottom half of a fraction and that describes the number of parts in the fraction.

Discrete Graph.. A graph that has discontinuous information that results in breaks in the display (the graph shows the temperatures only for weekdays and not for weekends).

Diagram.. A sketch used to illustrate an idea.

Digit.. A number.

Distribution ..Data that is organized by how often it occurs (i.e., a display of the numbers of different types of cars purchased in a particular year).

Distributive Property.. Describes that two numbers can be added together before being multiplied by a third number or each multiplied by that third number and then added and still produce the same answer i.e., $\{a(b + c) = ab + ac\}$.

Edge.. The place or line where two sides of a figure meet (i.e., the edge of a table).

Empirical A statement that is based upon observation and experimental evidence.

Estimates To give an approximate rather than an exact answer

Expanded Notation.. A way to write numbers that shows the place value of each digit. (i.e., $343 = 300 + 40 + 3$)

Exponent A numeral used to tell how many times a number or variable is multiplied by itself (i.e., $3^2 = 3 * 3$).

Extend (pattern) To continue a pattern or sequence with the same rules.

Exterior Polygon A figure with at least four sides, all of which are straight and with a share that has a least one angle going "into" instead of "out of" the shape.

Extrapolation Estimating the value of a number using the value of known numbers that precede it.

Face A flat surface of a three-dimensional figure.

Face Value The value assigned to each digit (0,1,2,3,4,etc.) used in our number system.

Factors The numbers that are multiplied to give a product (In $3 * 8 = 24$, the factors are 3 and 8).

Formula An equation that states a fact or rule ($lw = A$).

Fraction ... A number in the form $\frac{a}{b}$, where b is not zero

Frequency Table A listing of data that includes the number of times an item occurs.

Function A relationship between two items, one of which depends on each other (one tricycle has 3 wheels, two tricycles have 6 wheels, ...) that can be graphed and described algebraically.

Geometric Sequence A sequence of numbers in which the next term can be found by multiplying the current term by some number (given the number 3 and continuing to multiply by 2 gives the sequence 3, 6, 12, 24...).

Graph A pictorial representation of information or relationships between numbers presented on a coordinate plane used to represent data.

Greatest Common Factor .. Largest factor that two or more numbers have in common (GFC) (i.e., the GFC of 8 and 12 is 4)

Greatest Possible The measurement error which always results from rounding or estimating a number **error** measure to nearest specified unit of measure.

Histogram A particular kind of graph that compares two kinds of data for different time periods (the speed of female and male runners in each year between 1990 and 1996).

Hypotenuse The side of a right (90 degree) triangle that is across from the right angle.

Improper Fraction .. A fraction in which the numerator is greater than the denominator

Indirect Measure A measure found by using a formula or other strategy and not actually measuring something (i.e., finding the height of a tree without actually holding a ruler next to it).

Inductive Reasoning A particular type of logic which involves drawing conclusions from several specific, known facts and using them to make generalizations about other, similar situations.

Integers Positive and negative whole numbers.

Interpolation Estimating the value of a number using the value of known numbers on either side of the missing number.

Intersecting Lines Lines that cross and have exactly one point in common.

Inverse Operations Two operations that undo each other (i.e., addition and subtraction).

Justify To define or develop a postulate or theorem to explain and support a conclusion.

Least Common Multiple.. the smallest number for which two or more numbers are factors (i.e., the LCM of 3, 4, and 6 is 12)

Line of Symmetry A line that divides a geometric shape into two parts. Folding on the line of symmetry will produce two shapes that are the mirror image of each other.

Line Segments A part of a line having two endpoints.

Linear Equation An algebraic equation that describes or produces a straight line.

Manipulatives Tools, models, blocks, tiles, and other objects which are used to explore mathematical ideas and solve mathematical problems.

Mass The amount of "stuff" (atoms & molecules) inside solid objects. The weight of objects can vary depending on the location (the moon versus Earth), but the mass remains constant.

Mathematical Modeling Representing or showing mathematical ideas and relationships using objects, pictures, graphs, equations and other methods.

Matrices Plural of matrix, a way of displaying information in a grid form (a chart that lists the various models of cars for various years).

Mean In a collection of data, the sum of all the data divided by the number of data.

Measures of Central Tendency The mean, the mode, the median, and the geometric mean are commonly used.

Median The middle number (or the average of the two middle numbers when necessary) in a collection of numbers that are arranged in order from least to greatest.

Mixed Number.. A number that is equal to the sum of a whole number and a fraction

Mode The number that occurs most often in a collection of data.

Model (noun)... A representation of concrete materials, objects or drawings

Multiple The product of a whole number and any other whole number.

Multiplicative Inverses Two numbers whose product is one ($7 * 1/7 = 1$).

Net.. A two-dimensional representation of the surface of a three-dimensional figure that has been unfolded

Non-Standard Units Units of measure that are not fixed or set (handful, arm's length, stride).

Number line.. A one-dimensional picture in which the integers are shown as specially-marked points evenly spaced on a line. Although this image only shows the integers from -9 to 9, the line includes all real numbers, continuing "forever" in each direction. It is often used as an aid in teaching simple addition or subtraction, especially involving negative numbers.

Number Sentence... an equation or inequality with numbers

Obtuse Angle An angle with a measure that is greater than 90 degrees and less than 180 degrees.

Odd A whole number that has 1,3,5,7, or 9 in the ones place.

One to One Correspondence..The ability to match each member of one set to the member of an equal set

Operation The process of carrying out rules of procedure, such as addition, subtraction, differentiation.

Order of Operation The order in which a mathematical problem is solved which is generally to multiply and divide before adding and subtracting from left to right.

Ordered Pair A pair of numbers that gave the location of a place on a map or a graph (latitude and longitude).

Ordinal Numbers... A number indicating position in a series or order. The ordinal numbers are first (1st), second (2nd), third (3rd), and so on.

Parallel Lines Lines in the same plane that are always the same distance apart.

Patterns... A set or sequence of shapes or numbers that are repeated in a predictable manner

Perimeter The distance around a geometric shape.

Permutation An arrangement of items in which order is important (i.e., a list of the possible 1st, 2nd, and 3rd place winners).

Perpendicular Two lines that intersect to form right angles.

Pictograph A graph that shows numerical information by using picture symbols.

Place Value The value of a digit as determined by its position in a number (in the number "11" the one is worth either 10 or 1, depending on the position).

Plane Figures A geometric figure that is flat (one whose points are all in one plane).

Polygon A closed plane figure with straight sides (made up of line segments).

Polynomials The algebraic expressions with two or more terms (i.e., $5x + 2y + 3z$).

Power Another way to describe the exponent (i.e., $12^3 = 12 * 12 * 12$ or 12 to the 3rd power).

Precision of Measurement Tells how finely a measurement is made.

Prime Number A whole number greater than 1 that can only be divided evenly by itself and 1 (i.e., 17).

Prism A three-dimensional figure with triangular faces and two parallel, congruent faces.

Probability The number of favorable outcomes compared to the number of possible outcomes of an experiment.

Proper Fraction... A fraction whose numerator is an integer smaller than its integral denominator

Proportions A sentence that states two ratios are equal.

Pyramid A three dimensional figure with a flat base and triangular sides that meet in a point.

Pythagorean Theorem A relationship that occurs in every right triangle written $a^2 + b^2 = c^2$ and meaning that the square of the length of one leg added to the square of the leg of the other leg is equal to the square of the hypotenuse.

Radicals Another name for the roots of numbers such as the square root of 5 or the cube root of 17.

Range The difference between the greatest and the least numbers in a collection of data (i.e., the range of 2, 7, 13, and 17 is 15).

Ratio A pair of numbers that compare two quantities or describe a rate.

Rational Number Any number that can be written as a fraction (fractions may not have zero in the denominator).

Ray A part of a line that has one endpoint and extends endlessly in one direction (sunbeam).

Real Numbers All of the rational numbers which are all of the numbers that can be written as a fraction.

Reasonable Estimations... approximations based on mathematical reasoning that are within the desired degree of accuracy (i.e., $35+43=$ reasonable estimation would be 75 or 80 not 100 or 700)

Reflection The mirror image of a figure.

Regrouping In adding or subtracting, "carrying" and "borrowing".

Relationship A description of how particular numbers are connected.

Right Angle An angle that measures exactly 90 degrees.

Right Triangle A triangle in which one angle is a right angle (equal to 90 degrees).

Root A number that can be used as a factor a given number of times to produce the original number (i.e., the fifth root of 32 is 2 because $2 \times 2 \times 2 \times 2 \times 2 = 32$).

Rotation A transformation obtained by rotating a figure around a given point.

Rounding Numbers Expressing a number to the nearest one, nearest ten, nearest hundred and so on.

Sample Space A way to list all the possible results or outcomes for a probability experiment.

Scientific Notation A particular way of writing a number with the first number being written as a number between 1 and 10, with as many decimal places as necessary and the second number being 10 and the correct exponent (i.e., $7,923 = 7.923 \times 10^3$).

Secant A straight line intersecting the circle at two points.

Sequence A series of numbers that are predictable and can be extended using simple addition or subtraction (4, 7, 10, 13...).

Skip Count To count by multiples of a number (count by two, five, tens, etc.)

Slope The degree of steepness.

Solve To find all the solutions of an equation or other mathematical problem.

Standard Units Units of measure that have an accepted value like inch, cup, meter, and pound.

Standard Notation.. a number written with one digit for each place value in base ten; the most familiar way of representing whole numbers, integers, and decimals is standard notation (i.e., three hundred fifty six is 356)

Stem and Leaf Plot A method of organizing data for the purpose of comparison where the "leaf" is the number in the smallest place value and the "stem" includes the numbers in the larger place values.

Strategy A method or way of solving a problem.

Substitution Algebra Substituting or replacing something, a variable or an expression, in one equation with an equivalent expression from the other equation.

Symbolic Language Mathematical ideas expressed in a symbol or symbols.

Symmetry When an object can be folded in half to form two mirror image objects.

Tangent A straight line intersecting the circle at exactly one point.

Tessellation A covering of a plane with congruent shapes that exactly cover the area (tiling).

Theoretical Probability Counting the number of ways an event can happen using mathematical expectations.

Theory The belief about a particular concept, along with its proven facts and conjectures.

Three-Dimensional An object such as a cube that has a length, a width, and a height.

Translation Changing the position of a geometric figure by sliding it in any direction without any rotation or turn.

Transversal The name given to a line that intersects two or more other lines in a given plane.

Tree Diagram A method of finding all of the possible outcomes of an experiment by systematically listing the possibilities.

Triangular Numbers The numbers 1,3,6,10....are triangular because they can be expressed by employing the number of dots in successive triangular arrays of dots (this can be thought of as "stair-step numbers" or 1, 1+2, 1+2+3, 1+2+3+4, ...).

Two-Dimensional A flat figure such as a piece of paper. It has a length and a width, but not height.

Validate To give evidence that a solution or process is correct.

Variable A letter used to stand for a number in an expression or an equation.

Verify Proving a solution or process is correct.

Vertices The points where two lines come together (corners).

Volume The number of cubic units that fit inside a space figure.

Whole Numbers The set of natural numbers plus the number zero: (0,1,2,3,4.....).